

# Unexplored regions in QFT and the conceptual foundations of gauge theories

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June 2011

## Contents

<b>1</b>	<b>Introductory remarks</b>	<b>2</b>
<b>2</b>	<b>History of electrically charged fields and the problem of infraparticles</b>	<b>5</b>
<b>3</b>	<b>Lessons from 2-dim. infraparticles</b>	<b>13</b>
<b>4</b>	<b>Localization peculiarities of zero mass Wigner representations</b>	<b>19</b>
<b>5</b>	<b>Aharonov-Bohm effect for and violation of Haag duality</b>	<b>30</b>
<b>6</b>	<b>String-localization of charged states in QED and Schwinger-Higgs screening</b>	<b>39</b>
6.1	A note on massive strings . . . . .	50
<b>7</b>	<b>A perturbative signal of "invisibility" and "confinement"?</b>	<b>50</b>
<b>8</b>	<b>Resumé and concluding remarks</b>	<b>56</b>
<b>9</b>	<b>Appendix</b>	<b>59</b>
9.0.1	Modular group for conformal algebras localized in doubly-connected spacetime rings . . . . .	59

## Abstract

Massive quantum matter of prescribed spin permits infinitely many possibilities of covariantization in terms of spinorial (undotted/dotted) pointlike fields, whereas massless finite helicity representations lead to large gap in this spinorial spectrum which for  $s=1$  excludes vectorpotentials. Since the nonexistence of such pointlike generators is the result of a

deep structural clash between modular localization and the Hilbert space setting of QT, there are two ways out: gauge theory which sacrifices the Hilbert space and keeps the pointlike formalism and the use of stringlike potentials which allows to preserve the Hilbert space. The latter setting contains also string-localized charge-carrying operators whereas the gauge theoretic formulation is limited to point-like generated observables.

This description also gives a much better insight into the Higgs mechanism which leads to a revival of the more physical "Schwinger-Higgs" screening idea.

The new formalism is not limited to  $m=0$ ,  $s=1$ , it leads to renormalizable interactions in the sense of power-counting for all  $s$  in massless representations.

The existence of stringlike vectorpotentials is preempted by the Aharonov-Bohm effect in QFT; it is well-known that the use of pointlike vectorpotentials in Stokes theorem would lead to wrong results. Their use in Maxwell's equations is known to lead to zero Maxwell charge. The role of string-localization in the problem behind the observed invisibility and confinement of gluons and quarks leads to new questions and problems.

PACS: 11.10.-z, 11.15-q, 11.10Gh, 12.20.-m, 12.38.-t

## 1 Introductory remarks

Particle theory of the past century has led to a vast body of knowledge, but many theoretical problems about the conceptual foundations of these discoveries remained unresolved. Ideas as quark/gluon confinement have not been understood in terms of quantum field theoretical interactions. Even less ambitious looking problems as why and how the description of electrically charged particles in quantum electrodynamics (QED) requires a noncompact localization, whereas the Schwinger-Higgs screening counterpart of scalar quantum electrodynamics is pointlike-generated, still lacks good understanding.

To get a feeling for the dimension of the problem in relation to completely solved older foundational problems, compare this situation to the post Faraday-Maxwell but pre Einstein era of classical electrodynamics. Already 20 years after its discovery, *everything*, except the problem posed by the ether, was in place; and when Einstein removed the ether from its throne, no equation in Maxwell's theory and not even the Lorentz transformation had to be modified. Although the removal of the idea of an ether by Einstein's theory of relativity was an epoch-making event without which the emergence of quantum field theory (QFT) is not imaginable, classical field theory and in particular Maxwell's equation did not change their mathematical form. The action at the neighborhood (Nahewirkung) principle was fully established; only its connection with space and time was still awaiting drastic conceptual modifications.

The situation in QFT is very different; none of its fundamental equations for interacting fields and particles has been brought under mathematical control. In particular its best result, the renormalized perturbation theory, is known to diverge. Even after almost 9 decades there is simply no conceptually closed

part of QFT which can be compared with the closure of Maxwell's classical ED. The excellent conceptual understanding of quantum mechanics (QM) reached after less than 3 decades shows that an explanation has to go beyond the shared letter  $\hbar$  and operators in Hilbert space.

When people say that QFT, in particular the standard model (SM), explain an unprecedented amount of data, they refer to the power of prediction (sometimes post-diction) which results from combining calculations based on perturbative renormalization theory together with phenomenologically motivated assumption. This is indeed impressive, but it should not create the illusion that QED, not to mention the standard model (SM) into which it has been incorporated, has reached the conceptual maturity after more than half a century which Maxwell's theory attained already after only two decades. To say that quantum theories are inherently conceptually more opaque does not help and is not convincing since quantum mechanics (QM) in its epistemological and conceptual understanding does not lag much behind classical field theory. In fact as a result of its often counterintuitive structure, there is hardly any other area of physics which had received as much successful attention as QM. Rather we should admit that, although we have developed a detailed vocabulary within an impressive set of results obtained by a variety of computational tools, the main impact has been to impress ourselves; from a comprehensive understanding of the physical concepts of e.g. QED we are presently further away than the discoverers of renormalization theory considered themselves to be. Being aware about this state of affairs even with respect to our oldest and best studied QFT, it is less disquieting that the central problems behind what we were rather quick to call *gluon/quark confinement*, and which thereby acquired a kind of mental reality, persevered for more than 5 decades without any essential progress.

In view of the impressive progress (renormalization theory including Yang-Mills theories, dispersion relations, beginnings use of low-dimensional QFT as theoretical laboratories) which was sustained well into the 70ies, this raises the question about the reasons for the more than 4 decades lasting stagnation. Are we less intelligent, have the problems become too difficult, or has something gone wrong at an important conceptual crossing which forced particle theory into a dead end? This will not be the subject of this paper, we refer the reader to [1][38].

The present work is an attempt to bring some movement into a subject which, although outside the range of the above critical remarks, is still far removed from its closure. The only attempt with a similar aim was made by Mandelstam when he tried to formulate the QED interaction in terms of field strengths instead of point-like vectorpotentials. Our starting point is the recognition that massless higher spin  $s \geq 1$  representations exhibit a certain *clash between localization and the Hilbert space setting* of quantum theory. More specifically it is not a clash between the  $(m = 0, s \geq 1)$  representations and unitarity as such, but an incompatibility of Hilbert space positivity with the existence of pointlike generators with a prescribed covariance behavior. In such a situation there are two ways out: to cede on the side of Hilbert space or on the side of pointlike localization.

The first path leads to the gauge-theoretic formulation and is inexorably related to indefinite metric spaces, whereas the second setting deals with string-localized covariant potentials which interact with matter fields and transfer their noncompact localization to the latter. The stringlike localization of matter fields (Maxwell charges, Yang-Mills charges) belong to what one could call the *non-local gauge-invariants*. But this is only a word since the ghost formulation of gauge theory is not capable to construct such objects; it is simply not made for addressing any physical object unless it is pointlike generated.

The gauge theoretic formalism has the advantage that its perturbative formulation makes intimate contact with the formalisms of constraint classical field theory as the BRST setting or the Batalin-Vilkovisky formalism which permit to represent the constraint solution space in cohomological terms. Although the protagonists of these cohomological formalisms for handling constraints are quantum physicists, the idea to solve problems by first enlarging the number of fields (by ghosts and other unobservable objects) and then playing on cohomological properties in order to take care of constraints is entirely classical<sup>1</sup> and draws its importance from the fact that the quantization approach (as an (artistic) parallelism between classical and QFT) requires a good preparation on the classical side. As the name "ghost" indicates, the enlarging objects which are natural in their classical context lead to a breakdown of the Hilbert space in attempts to quantize this formalism; but at least perturbatively the compatibility with a Hilbert space representation *after the cohomological descent* can be established in terms of successful recipe of "quantum gauge invariance" [2].

There is however a high prize to be payed on the side of QT. Whereas the intermediate abandonment of the Hilbert space can be accepted as a provisional perturbatively convenient tool, the problem that the physically most important electrically charged operators cannot be constructed in this perturbative setting (because they are string-like generated, and classical Lagrangians theory does not go beyond pointlike fields) is a more serious obstacle and poses a challenge to look for something else.

This consists in the use of non-pointlike free fields which have no Lagrangians<sup>2</sup>, a step which places the localization property on which QFT is founded back into the center stage. Localization is important in QM and QFT and this carries the danger to overlook the enormous conceptual-mathematical difference in their localization structures namely that of the (Newton-Wigner<sup>3</sup>) Born localization in QM and the "modular localization" (quantum causal localization) for QFT. Elsewhere it was shown how the confusion between the two has led particle physics into its still ongoing crisis [38]. The topic of this paper is more upbeat: the modular localization will be used to shed new light on remaining problems

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<sup>1</sup>Classical covariant pointlike potentials are perfect objects of a classical constraint formalism. The Hilbert space requirement and the resulting delocalization is a pure quantum effect and only happens for "potentials" in  $(m=0, s \geq 1)$  representations.

<sup>2</sup>There are even many spinorial free fields which are not of the "Euler-Lagrangian" kind. The latter are only needed in functional integral inspired constructions.

<sup>3</sup>The addition of these names serves to remind the reader that the suitably adapted localization connected with probabilities also exists in the relativistic setting. In relativistic QM it is the only localization [29].

of gauge theory whose solution have a good chance to take the unfinished SM out of its 40 year conceptual stagnation.

More concrete: it will be shown that modular localization resolves the clash between pointlike localization and Hilbert space structure which affects vector/tensor potentials of zero mass ( $m = 0, s \geq 0$ ) Wigner representations and the resulting covariant semiinfinite string-localized potentials in a QED-like interactions delocalize charged particles without affecting the pointlike localization of field strength. This paper falls short of a systematic perturbation theory; partially because important conceptual steps, as the generalization of the Epstein-Glaser iteration, have yet to be accomplished.

Since the historical roots are not that well known anymore among novices of QFT, the next section will review some of the old problems however with modern hindsight. Often problems, which at the time were too difficult to be solved in the realistic context, have been analyzed by looking at their analog in the "theoretical laboratory" of two-dimensional QFTs. For the problem at hand this will be commented on in section 3. The following section 4 contains the representation theoretical setting. Together with modular localization THIS leads to the interaction-free string-localized (s-l) covariant potentials. Section 5 shows that already in the absence of interactions the use of pointlike indefinite metric vectorpotentials is a risky business since in contrast to the Hilbert space compatible s-l potentials they may lead to incorrect results of which the absence of the well established quantum Aharonov-Bohm effect (the violation of Haag duality for toroidal regions) is an illustration.

In section 6 the main issue is to give perturbative arguments about the transfer of delocalization from potentials to charged fields which thereby loose their pretended pointlike nature. This section also presents the Higgs mechanism in a different more physical setting of Schwinger-Higgs screening which leads to the re-localization of the nonlocal aspects of Maxwell charges in analogy to Debye's screening in QM which converts the Coulomb potential into an effective short range interaction of the Yukawa kind. The new aspect is that screening in QFT is not just a vanishing of the global charge operator on physical states, but also involves the return from s-l to point localization. In the last section some more speculative points of view about confinement/invisibility of states in connection with nonabelian gauge theories are mentioned before some of the high points are resumed in the concluding remarks.

## 2 History of electrically charged fields and the problem of infraparticles

The gradual improvement of the understanding of the local quantum physical aspects of electrically charged fields and their associated particles is one of the most fascinating projects of QFT. Even after a resounding success in describing observed data dating back more than 70 years, is not anywhere near its conceptual closure. It leads to a particle-field relation which is far more subtle than the

standard textbook case of an energy-momentum spectrum with mass gaps. In the latter case one obtains in a well-known manner (via large time asymptotes) free fields; together with the concomitant assumption of asymptotic completeness<sup>4</sup> one then finds that the Hilbert space of such a theory has the form of a Wigner-Fock space.

The appearance of zero mass particles as such do not necessarily cause a breakdown of these scattering results. As long as the particle states in the presence of interactions manage to preserve their particle aspect i.e. are *not* "sucked" into a continuous part of the mass spectrum (example the particle properties of the nucleon are not affected by the interaction with zero mass pions) the results of scattering theory continue to be valid, even though there are no mass gaps.

But interactions of charged particles with photons are not covered by this kind of scattering theory. Already the quantum mechanical Coulomb scattering leads to problems with the large time behavior of amplitudes, although in this case the multiparticle tensor structure of the Hilbert space remains unaffected and the large time limits converge after removing a logarithmic time dependent phase factor [13]. In QED the failure of time-dependent scattering theory is more severe, in fact it is inexorably related to a breakdown of Wigner particle states and the absence of a Wigner-Fock structure of the QED Hilbert space i.e. it is not limited to multi-particle scattering amplitudes but even changes the nature of one-particle states themselves. Instead of the mass shell contribution in the Källen Lehmann twopoint-function one finds a branch cut whose strength is restricted by unitarity and which leads to a vanishing LSZ limit making it impossible to define a nontrivial scattering matrix. On the other hand in perturbation theory the restriction to the mass shell causes infrared divergencies which, unlike the Coulomb scattering in QM. *cannot be repaired* on the level of scattering amplitudes but only by passing to infinite soft photon-inclusive cross sections with infrared cutoffs in intermediate steps and a dependence on the method by which the cutoff is introduced and afterwards compensated. In contradistinction to the elegant spacetime representation of the scattering amplitudes in the LSZ formalism there is (yet?) no known spacetime representation.

It is the principle aim of this paper to establish that these unusual momentum space properties are caused by the string-localization of vector potentials which, figuratively speaking, feed their string localization through the interaction to the charged matter fields. Whereas for the potentials themselves the effect of string localization has no direct physical consequence as long as there is no direct coupling between them (abelian gauge theories) and the relation to the pointlike field strength remains unaffected, the charged particles, which were described by pointlike matter fields in zero order perturbation order, become semiinfinite string-localized in a way which cannot be undone in any operational way (differentiation or other lin. operations). This weakening of localization of quantum Maxwell charges is behind the *infraparticle* concept. But before we

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<sup>4</sup>All presently known constructions are based on the two-dimensional "bootstrap-formfactor setting" (factorizing models). For those constructions which start from particles and introduce fields through their formfactors [39], the completeness property is built into the construction.

get there we will follow briefly the historical path.

Problems with the application of standard scattering theory to QED were noticed quite early since scattering theory in one form or other is one of the oldest tools in QT<sup>5</sup>; the research on infrared divergencies begun in a 1934 paper by Bloch and Nordsiek in which scattering of charged particles and photons was analyzed in a simplified model of QED [3]. The important conclusions, namely that although the number of photons in the infrared is infinite, the emitted energy and angular momentum remains finite, as well as the message that, in agreement with the vanishing of the LSZ scattering amplitudes, one has to sum over all infinitely many infrared photons up to a certain energy resolution  $\Delta$  determined by the measuring device in order to obtain a non-vanishing scattering probability (inclusive cross section<sup>6</sup>), can already be found in this early paper. Later perturbative covariant calculations in QED succeeded to confirm and improve these results [4].

If QFT would be limited to the finding of successful recipes, then this formalism, which proceeds "as if" charged particles would be Wigner particles and the LSZ asymptotic behavior would be valid modulo some infrared imperfections which can be repaired similar to the removal of exponential logarithmically diverging long distance factors in Coulomb scattering. But the infrared trouble is not limited to scattering amplitudes, its course is the breakdown of Wigner's one-particle structure and the possibility to describe QED in a Wigner-Fock space. Electrically charged quantum matter is semiinfinite string-localized in a very sense, the infinite string has wiped out the one particle pole  $p^2 = m^2$  and replaced it by a more complicated mass spread which has no characterization in terms of just two invariants  $m$  and  $s$ . For many pragmatic minded particle theorists the compensation of infrared divergencies on the level of soft photon inclusive cross sections would have been the end of the story. Fortunately theoretical physics was never pragmatic in this extreme sense, since one knows that suppressed conceptual and philosophical questions always return later on with a vengeance.

In this paper it will be shown that the infrared divergences are the *result of the presence of string-localized vectorpotentials* in the zero order interaction density. Their most important role is to *delocalize charge fields* with which they interact and as a result to lead to the phenomenon of the charged "infraparticles" which results in a continuous mass spread accumulated at the lower edge of a continuum. The effect on the string-localized vectorpotentials on themselves on the other hand remains very mild, even in the presence of interaction the associated field strength stay pointlike and the scattering theory for photons only remains close to the conventional setting [28]. This behavior occurs for all zero mass,  $s \geq 1$  potentials but not for  $s \leq 1/2$  which includes the aforementioned zero mass meson to nucleon coupling. The the reason behind the appearance of infraparticles is the semiinfinite string localization of the vectorpotentials which

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<sup>5</sup>The Born probability concept was first introduced in the setting of Born's scattering approximation, the extension to Schroedinger wave functions came later.

<sup>6</sup>In agreement with the vanishing of the LSZ amplitudes, one obtains vanishing of the probabilities in the limit  $\Delta \rightarrow 0$ .

the interaction transfers to the charged particles and the dissolution of the mass shell of the charged particle into the photon continuum is the momentum space consequence. In the presence of selfinteractions between vectorpotentials (e.g. Yang-Mills) there are no linearly related field strengths and the rules for change of the string localization becomes more complicated (interaction-dependent).

According to the previous remarks the two-point function of a charged field<sup>7</sup> in interaction should reveal a behavior at the mass shell which is different from a delta function. Unitarity imposes a strong restriction on the two-point function. For example derivatives of delta functions are excluded, since they are not positive measures; a moment of thought suggests that near  $p^2 \approx m^2$  the Kallen-Lehmann function should have an anomalous (coupling-dependent) power and behave as  $\theta(p^2 - m^2) (p^2 - m^2)^{f(e)} g(p^2, \alpha)$  which for vanishing coupling (fine structure constant)  $\alpha \rightarrow 0$  approaches the mass shell delta function and for finite  $e \neq 0$ , for reasons of representing a positive measure, is milder than the delta function. Physical QED (i.e. not its indefinite metric version) leads to a spontaneous breaking of Lorentz invariance in its charged sectors, but since in perturbation theory the calculation of correlations involving physically charged fields is prohibitively difficult, the normal practitioner does not see such things.

In the later part of the present work it will be shown that there is an additional vector in the problem which in principle also enters the infrared powers. This state of affairs (a milder mass shell singularity than a delta function) would immediately lead to a vanishing LSZ limit, which accounts for the observation by Bloch and Nordsiek about the vanishing of scattering leading to a finite number of photons after the compensation of the infrared cutoffs have taken place.

Since QED is hard to control, physicists began in the 60s to look at simple soluble two-dimensional *infraparticle models* which exhibited the expected coupling-dependent power behavior, similar to that which appeared in the scattering formulas of YFS [4] after summing over leading logarithms in the infrared photon cutoff. A common feature of those two-dimensional infraparticle models is the presence of an exponential zero mass two-dimensional Bose field factor in the generating field [5]. What was still missing was a structural argument that electrically charged particles are really infraparticles in this spectral sense, as well as a conceptual basis for a new scattering theory which does away with cutoff tricks and, as the standard LSZ or Haag-Ruelle scattering theory, only uses spacetime properties of correlation functions.

In QFT the overriding principle is causal localization and, thinking of the relation between energy positivity and localization (sections 3,4), it is in a certain sense the only one. This principle has a variety of different physical manifestations and the main problem of understanding a model of QFT consists in *finding the correct structural arguments which reveals the connection between the perceived properties of a model and their explanation in terms of causal*

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<sup>7</sup>With charged field we always mean the physical charged field, but it is not necessary to add this since the Lagrangian matter field  $\psi$  in the indefinite metric setting has neither electric charge nor has its localization any physical significance.



localization.

The role of localization can be exemplified in two physically very important cases (section 5), the string-localization of electric charges and the Schwinger-Higgs charge screening as a kind of "re-localization" process, leading to a vanishing charge (a loss of the charge superselection) causing a breaking of the charge symmetry. In the case of the before mentioned two-dimensional infraparticle models this consists in the realization that the complex zero mass Bose field is really a semiinfinite string-like localized field (next section) and it is this non-compact localization behavior which is at the root of the dissolution of the mass shell delta function into a cut type singularity. Computations may be easier in momentum space, but the deeper conceptual insight is always in the spacetime setting.

Suggestions that electrical charge-carrying fields as the interacting electron-positron field have necessarily a noncompact extension entered the discussion quite early; practically at the same time of the Bloch-Nordsiek work Pascual Jordan [6] started to use the string-like formal presentation of the physical charge field which, since this also was on Dirac's mind and is also often linked with the later work by Mandelstam<sup>8</sup>, will be referred to as the DJM presentation of a charged field.

$$\Psi(x; e) = \text{"}\psi(x)e^{\int_0^\infty ie_{el}A^\mu(x+\lambda e)e_\mu d\lambda}\text{"} \quad (1)$$

$$\Phi(x, y; e) = \text{"}\psi(x)e^{\int_0^1 ie_{el}A^\mu(x+\lambda(x-y))(x-y)_\mu d\lambda}\bar{\psi}(y)\text{"} \quad (2)$$

Gauge invariance not only suggested that physical<sup>9</sup> electrically charged fields have a noncompact localization with the semiinfinite spacelike string (1) being the tightest (least spread) possibility, but also that charge-neutral pairs are necessarily interconnected by a "gauge bridge" (2). The methods of local quantum physics permit to show the infinite extension of charges on the basis of a rigorously formulated quantum Gauss law (next section), but the above formula has no conceptual or computational preferential status and is not distinguished by renormalization theory; in fact *a physical (Maxwell) charged field does not appear at all within the standard (Gupta-Bleuler, BRST perturbative formalism [19][18].* This zero Maxwell charge effect in the presence of ghosts is an analog of the zero magnetic Aharonov-Bohm effect in such a setting (section 5). Formally the correct result would be obtained after imposing the BRST invariance but nobody knows how to do this for nonlocal expressions; the classical BRST formalism is only made to construct local observables. In a Hilbert space description with string-like potentials these effects are correctly described and the problem is shifted to the renormalization theory in the presence of string-localized free fields.

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<sup>8</sup>Mandelstam's [16] use of line integrals over field strength is certainly an early attempt to preserve the Hilbert space structure by easing on pointlike locality. But the central position of causal localization in QFT was not yet fully recognized.

<sup>9</sup>The pointlike formal matter fields which enter the Lagrangian and field equation of gauge theories are auxiliary quantities which act neither in a Hilbert space nor does their localization have a physical significance.

The rigorous definition of these charged fields in renormalized perturbation theory is a nontrivial problem (the reason for the quotation marks in (1)). Steinmann had to develop a separate perturbative formalism only for defining the renormalized physical DJM charged fields [7]. The gauge setting leads to a nice picture suggesting the semiinfinite string-localization of charged fields, but, just as the abstract argument based on the quantum Gauss law, it does not really explain the origin of this weaker localization in terms of the form of the interaction. In the standard gauge setting the latter is pointlike i.e. it looks like any other local interaction. This shifts the problem of physical localization to a level where definitions are cheap but their constructive use turns out to be difficult. The perturbation theory of QED in the BRST setting follows standard rules, but the calculation of BRST invariant correlations is prohibitively difficult if nonlocal operators enter the search; even applied to local operators it is difficult since the BRST transformations resembles that of a nonlinear acting symmetry. In fact most of the papers present the formalism, but fall short of calculation BRST invariant correlations of charged objects.

Needless to add that for Yang-Mills interactions even the first step is beset by inexorably intertwined ultraviolet-infrared divergencies which impede the execution of the renormalization program in any covariant gauge. Ultraviolet divergencies without intermingled infrared problems can be renormalized which expresses the fact that in a more intrinsic approach which avoids the quantum mechanical aspects of the standard (cutoff, regularization) methods in terms of a more intrinsic (Epstein-Glaser) approach (which takes better care of the intrinsically singular structure of quantum fields resulting from vacuum polarization). A particular elegant formulation is the "algebraic adiabatic limit" method [49] which is based on the dissociation of the algebraic structure from that of states. The latter step is the analog of the DHR superselection construction but the existing theory [28] is not applicable to Maxwell charges. Infrared divergencies which hide physical localization problems exist for all theories in which massless higher spin  $s \geq 1$  potentials participate in the interaction, independent of the presence of ultraviolet problems. Their control is not a matter of some mathematical-technical adjustments but rather a major revision of the physical setting. The string-like formalism leading to a distributional directional dependence allows to separate them.

The fact that the fundamental electrically charged fields cannot have a better localization as semiinfinite stringlike, has of course (at least implicitly) been known since the DJM formula (1), but the infrared problem showing up in scattering and the problem of semiinfinite string-localized charged fields have for a long time not been linked together. It is one of the aims of this paper to show that they represent two sides of the same coin.

The standard gauge formalism is, apart from some conceptual problems<sup>10</sup>, very efficient in setting up a perturbative formalism for *local* gauge invariants. As mentioned one of the reasons why this formalism is not suitable to formulate

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<sup>10</sup>Different from non-gauge QFT the Hilbert space for the local observables is not defined at the outset but rather results from an auxiliary indefinite Hilbert space through a cohomological construction.

the problems behind the infrared divergencies is that it does not give any clue how to deal with electrically charged operators; they are just not part of any existing perturbative formalism. Attempts to attribute physical significance to the Dirac spinors in the indefinite metric formalism have ended in failure; there is simply no subspace on which the Maxwell equations can be defined and on which states with a nontrivial electric charge can be introduced, the pointlike  $\psi$ 's carry no Maxwell charge and their pointlike localization is a fake [19][18].

If the underlying philosophy of local quantum physics (LQP), which led to the construction of charge superselection sectors solely from the structural data of observables, would also apply to Maxwellian charges, one should be able to reconstruct the charge neutral bilocals with gauge bridges (2) from the algebra of charge-neutral pointlike local bilinears. For globally charged fields (in which case there are no connecting gauge lines) this has been shown [8] in a model, but for Maxwellian charges there are yet no mathematically decisive result [9]. But gauge bridges are much more removed from what can be reached on the colloquial radar screen of standard gauge theory than string-like potentials.

We propose a new approach for theories in which string-localized potentials associated to zero mass finite spin representations interact with massive quantum matter. This includes in particular models of gauge theories. The new setting is designed to incorporate the physical charged fields into the perturbative formalism. Figuratively speaking, the potentials which have a rather harmless string dependence pass the string-localization onto the massive field where it keeps piling up in perturbation theory and (in contrast to the potentials themselves) becomes *irremovable by any linear operation* already in the lowest nontrivial order (section 5).

The starting point is a combination of Wigner's representation theory for ( $m = 0, s \geq 1$ ) with modular localization. The result is that, whereas the various possible "field strengths" are pointlike covariant wave functions (or pointlike quantum fields in the functorial associated "second quantization"), their "potentials" are semiinfinite stringlike localized objects. The stringlike potential setting solves also another problem. For  $s \geq 1$  the use of field strength does not allow interactions which are renormalizable in the sense of power-counting, but this interdiction does not hold for couplings with stringlike potentials; there always exist polynomial interactions of maximal degree 4 in terms of stringlike potentials of short distance scaling dimension  $d_{sc} = 1$  which are renormalizable by power-counting. Whether desired interactions (as the Einstein-Hilbert action for  $s=2$ ) are among those is a separate question.

Textbooks and review articles on gauge theories in general (and on QED in particular) often create the impression that they represent the best understood QFTs. It is certainly true that they are the physically most important models and they lead to rich perturbative calculations. It is also true that in the semiclassical form of quantum theory in external gauge fields they led to deep mathematics and broadened the level of mathematical knowledge of several generations of theoretical physicists, but those geometrical structure (fibre

bundles, cohomology), contrary to a widespread opinion<sup>11</sup>, are not the kind of structures which are important for the solution of the above quantum problems. Nevertheless both the (semi)classical and the local quantum physical aspects of these models are both very rich in their own right. They are certainly the most interesting theories, especially in the setting which we are going to present.

As mentioned, the models which are conceptional well-understood are those which have a mass gap and hence fall into the range of applicability of the LSZ/Haag-Ruelle scattering theory; but after the acceptance of the standard model those models have been of a lesser observational and conceptional interest.

There are also theories which, at least at the outset, have a somewhat hidden mass gap than those which served as illustrations of the LSZ formalism. These are the models which owe their mass to the Schwinger-Higgs screening mechanism and play an important role in the standard model. They involve massive vectormesons, but more generally all models which rely on interactions with higher ( $\geq 1$ ) spin objects are interesting because they pose a challenge to renormalization theory, as it will be shown in section 3.

Schwinger's contributions to the screening idea has been forgotten, therefore some historical remarks are in order. At the end of the 60s Schwinger envisaged the possibility of a "screened phase" in actual (spinor) QED in which the photon becomes a massive vectormeson. Since I could not find any convincing argument, he looked at  $d=1+1$  massless QED (the Schwinger model) where he could exemplify his screening idea [10]; the model led to a vanishing (totally screened) charge showing also that the screening mechanism is not related to the Goldstone spontaneous symmetry breaking since the latter has no realization in  $d=1+1$ . The screened version of scalar QED in  $d=1+3$  is identical to the model proposed by Higgs [11]. A screening mechanism for  $s=1/2$  quantum matter in  $d=1+3$  does not exist, at least not in perturbation theory. Therefore one always needs to start with scalar QED or with couplings of Yang-Mills fields to complex scalar fields in order to obtain massive vectormesons interacting with Schwinger-Higgs screened (real) scalars. If other fields as  $s=1/2$  Dirac fields are coupled to massive vectormesons obtained from screening, the only charge which survives is the global charge carried by Dirac spinors whereas the Maxwell charge and its nonabelian counterpart vanishes.

We will limit our main attention to two models namely scalar QED with its delocalized (string-localized) charged fields, and associated infraparticles, and the Higgs model, which in some sense to be made precise may be viewed as the result of "charge screening" leading to mass gaps and a return to pointlike locality. With the exception of this "mass-generating" Schwinger-Higgs screening mechanism, all gauge theories contain strongly delocalized objects, namely visible Maxwell type charges or invisible gluons/quarks.

In the following it will be shown that the infrared divergence properties and their first cure in the famous Bloch-Nordsieck [3] scattering model and Jordan's stringlike (1) formula which represents a physical electrically charged field, are really two different sides of the same coin; they are both consequences of the fact

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<sup>11</sup>They did however raise the level of mathematical sophistication of particle physicists.

that certain quantum objects by their intrinsic nature do not permit pointlike generators but rather possess only noncompact localized generators. As the family of arbitrarily small double cone<sup>12</sup> localization (the natural shape of a compact causally closed region) is pointlike generated, and the tightest causally complete noncompact localization, namely a thin a spacelike cone, has as its core a semiinfinite spacelike semiinfinite string.

It may be more than a curiosity that both observation, the one on infrared divergencies in scattering of charged particles, and the stringlike DJM formula were made at practically the same time. Couldn't it be that there is an aspect of the subconscious in the Zeitgeist? Hard to decide because Jordan used the DJM formula mainly for deriving an algebraic magnetic monopole quantization in the same year that Dirac presented his geometric derivation. Historical speculations aside, the understanding of why these two observations belong together is more recent and constitutes a strong motivation for the present work.

The content is organized as follows. In the next section we review the two-dimensional infraparticle models and show that all of them are string-localized (half-space-localized). The section also contains the known rigorous statements about higher dimensional infraparticles.

Section 3 presents the theory of string-localized potential fields starting from Wigner's representation theory and illustrated in more detail for  $s=1, 2$ .

The last section presents some rudiments of a new formalism, in which instead of enforcing pointlike potentials and paying the prize of being temporarily thrown outside quantum physics (by the occurrence of indefinite metric), one rather sticks to a setting in which one confronts the true stringlike localization instead of working with a simpler and familiar pointlike formalism which, apart from local BRST invariants does not describe the true localization of some of the most important physical objects a charged fields.

### 3 Lessons from 2-dim. infraparticles

The Bloch-Nordsiek treatment of the infrared aspects of scattering of electrically charged particles and its extension to full perturbative QED in the work of Yennie, Frautschi and Suura [4] led to the notion of infrared finite *inclusive cross sections* in which infinitely many "soft" photons are summed over. This successful recipe did however not answer certain important conceptual questions. The quantum mechanical treatment of Coulomb scattering also leads to an infrared problem, namely a scattering theory of charged particles in which for large times a logarithmic time-dependent phase factor prevents the asymptotic convergence for large times [13] and where the remedy is either the removal of this factor from the amplitudes or passing directly from amplitudes to probabilities. In this case one knows in spacetime dependent terms what one is doing, however a manipulation in momentum space as in YFS is a recipe and only gains a conceptual status one finds a spacetime explanation for what one is doing.

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<sup>12</sup>A spacetime double cone is the unique kind of spacetime region which is compact, simply connected and causally complete.

So the question arose: is the QFT scattering of charged particles similar to the quantum mechanical Coulomb scattering in which one particle states and their n-particle tensor-products continue to exist in the Hilbert space of the theory and only the large time asymptotic convergence towards n-particle states is modified by infrared factors, or is there something more dramatically happening in the QFT scattering of charged particles?

The field theoretic phenomenon of vacuum polarization in the presence of interactions leads to the mutual coupling of all channels as long as they are not separated by superselection rules. Since relativistic scattering theory treats one-particle states and multiparticle states on the same footing, one would expect that, different from the Coulomb scattering, radical changes in the scattering concepts can not happen without a major modification of the particle concept, so that even a charged one-infraparticle state cannot be described as a kinematical object in terms of Wigner's irreducible representation theory. Since, as most QFTs in  $d=3+1$  which have no spectral gaps, QED is still outside mathematical-conceptual control, it has been useful to look for analogs in the more accessible two-dimensional "theoretical laboratory". QED shares with all other renormalizable theories in any dimension the divergence of the perturbative series so that great care has to be applied in drawing structural conclusions.

The exponentiation of leading logs to a power behavior in the YFS work suggested to consider models which contain a zero mass exponential Bose field  $\Phi^{13}$ , which in  $d=1+1$  has formal dimension zero and therefore leads (still formally) to logarithmic correlation functions of exponential operators with an anomalous operator dimension

$$\dim e^{i\alpha\Phi(x)} \simeq \alpha^2 \quad (3)$$

$$\mathcal{L}_{int} = \alpha \partial_\mu \Phi \bar{\psi} \gamma^\mu \psi, \quad \psi = \psi_0(x) : e^{i\alpha\Phi(x)} : \quad (4)$$

Historically the first use of a Lagrangian model as an "theoretical laboratory for the infrared" [5] was a zero mass scalar meson coupled to a massive nucleon via a derivative coupling (4) but actually this exponential should be called the Jordan model [6] since Jordan invented it at the same time of the Bloch-Nordsiek papers and his line integral presentation of gauge invariant charged fields (1) though for a very different purpose<sup>14</sup>. The resulting "interacting" Dirac spinor (4) leads to a Kallen-Lehmann representation in which, instead of the one-particle mass shell delta function, one encounters a cut which is starting at the position of the mass and for  $\alpha \rightarrow 0$  converges in the sense of distributions to the mass shell delta function. The strength of this cut (the power) is bounded by unitarity since the latter forces the K-L weight to be a measure. The free Dirac field  $\psi_0$  is a formal auxiliary object; in the autonomous Hilbert space constructed via the

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<sup>13</sup>Interestingly enough, this was also a model used by Jordan though not for the present purpose but rather for developing his ill-fated "Neutrino theory of light".

<sup>14</sup>At that time physicist thought that, as in QM, one can study particle phenomena in low dimensions and then simply generalize to higher ones. But from Wigner's work on particles we know that the representation theory of the Poincare group is very much dimension dependent and hence to infer from the chiral Bosonization-Fermionization relation a "neutrino theory of light" is really far-fetched [6].

GNS reconstruction from the Wightman functions of  $\psi, \bar{\psi}$  spinor fields there is no free  $\psi_0$  field and the unitary representation of the Poincaré group does not contain an irreducible component corresponding to a discrete mass.

Consistent with this structure is the observation that the LSZ large time asymptote vanishes i.e. instead of the standard incoming/outgoing free fields one obtains zero. This is so because the infraparticle singularity is too weak in order to match the dissipative behavior of particle wave function; in perturbation theory one encounters however the typical well-known logarithmic infrared divergencies. The non-perturbative model of Bloch and Nordsieck, as well as the summing up of leading terms in the YFS work, lead to vanishing emission amplitudes for the emission of a finite number of photons, which, as mentioned previously, is in agreement with the vanishing of the LSZ limit as a result of the softening of the mass-shell singularity mentioned before.

The field  $\Phi$  (4) has infrared properties which prevent it from being a Wightman field, since it cannot be smeared with all Schwartz test functions but only with those whose total integral vanishes. However the exponential is again a bona fide Wightman field if one imposes on the Wick contraction rules the charge superselection rule i.e. if one assigns to the exponential the charge  $\alpha$  and to its adjoint the charge  $-\alpha$ , so that the only products of fields with total charge zero have nonvanishing vacuum expectations.

The selection rules would follow from the exponential of a massive spinless field in the massless limit by imposing the condition that the fields are renormalized with appropriate powers of the mass in such a way that none of the correlations becomes infinite. This requirement is well-known to lead to the vanishing of all correlations of exponential fields except those involving charge neutral products. In the above derivative model (4) this local  $\alpha$ -selection rule is masked due to the presence of a second *global* charge conservation of a complex Dirac spinor.

For our purposes another method, which shows that the exponential field is *string-localized*, leads to more physical insight. It starts from a chiral current which is a well-defined quantum field and defines the exponential field as an exponentiated integral of the current over a *finite* interval followed by the spacetime limit which takes one of the endpoints to  $+\infty$  infinity in order to lessen the influence the compensating charge has on the physics of isolated localized charges. The correlation function only remain finite if the all the compensating charges at infinity and hence all the finitely localized charges add up to zero. This model indicates that charges, that there are certain charges which owe their existence to string localization and the claim is that this simple exponential Boson model illustrates certain features which in the realistic context of QED are much harder to demonstrate.

This method reveals that the resulting operators are localized along a string, namely the semiinfinite interval  $[x, \infty]$  where  $x$  is the endpoint which has been

kept fixed. So formally the  $\Phi$  in the exponential should be viewed as

$$\Psi_\alpha(x) = e^{i\alpha \int_x^\infty j(y)dy} \quad (5)$$

$$\Psi_\alpha(x, x') = e^{i\alpha \int_x^{x'} j(y)dy} \quad (6)$$

The main difference to (1), apart from the spacetime dimension is that the current is a physical massless field whereas the pointlike vectorpotential is not. It is quite easy to show that all chiral correlation functions, including the charge superselection rule, can be obtained from string-connected bilocals as defined in the second line (5). The  $\alpha$ -charge has similarities with a Maxwell charge in that it is locally generated. In spinor QED there is besides the Maxwell charge also the global charge<sup>15</sup>. But, as will be seen later, a Schwinger-Higgs screening where both the Maxwell charge also the notion of global charge as well as the charge which comes with a complex field disappear is only possible with scalar complex fields i.e. scalar QED. Needless to add, the zero chiral field plays a special role; it does not really exist rather what is behind it is a string-localized object which only makes sense in the exponential form.

String localized fields, as those exponentials, are outside the standard Wightman field theory in that they neither commute nor anticommute<sup>16</sup> for spacelike separations between endpoints  $x$ . In chiral conformal field theory as (5) the string localization is visible in *plektonic* (in the above abelian case *anyonic*) commutation relations representing braid group statistics. Just looking at the problem as a formal massless limit of a two-dimensional exponential would not reveal that behind the infrared divergences there is a transition from point-to string-like localization. The global Dirac charge and the local charge carried by the exponential line-integral in the above derivative model (4) lead to identical selection rules and have their counterpart in QED where the local charge is referred to as the Maxwell charge. In higher dimensions the string-like localized nature of charged fields is more easily perceived, since fields  $\Psi(x, e)$  where  $e$  is the direction (spacelike unit vector) of the semiinfinite spacelike string  $x + \mathbb{R}_+ e$  have a Lorentz transformation law in which the  $e$  participates, whereas in  $d=1+1$  there is no such variable direction. As usual for spontaneously broken symmetries, the Lorentz transformation on  $d=1+3$  charged fields exists only as an algebraic automorphism (which also acts on the string direction) which cannot be globally unitarily implemented.

The two-dimensional infraparticle models of the 60s and 70s have been recently re-discovered in order to illustrate a proposal in the setting of effective QFT called "unparticles". It is unclear how those models can illustrate two infrared concepts unless they are the same. Unfortunately the authors have only

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<sup>15</sup>"Global" in this context does not mean that there are no local properties or consequences. In the DHR theory [28] of superselected charges and inner symmetries, the global spinor charge is reconstructed via the (representation-theoretical) "shadow" it imprints on the charge-neutral local observables.

<sup>16</sup>For the particular value of the charge  $\alpha = 1/2$  (it depends on the normalization of the current  $j$ ) one obtains the massless  $d=1+1$  free Weyl spinor. For this reason the moving between  $\Phi$  and this spinor  $\psi$  has been called fermionization/bosonization, a terminology which is not completely correct since both live in different charge sectors.



sketched in vague perturbative momentum space setting what they require of unparticles. Apparently they believe that the Bloch-Nordsieck and YFS work only affects the scattering amplitudes but leaves particle states intact similar to Coulomb scattering in QM. But this is not true, behind the infrared problems in QED there are string-localized infraparticles and not just long ranged interaction potentials leading large time exponential logarithmic factors which are easily taken care of [13]. The unparticle proposal is based on an insufficient appreciation of the radicality of infraparticles. Writing down something in momentum space and naming it "effective QFT" without giving a hint what one wants in terms of localization is not revealing much.

The concept of infraparticles is not explained by just pointing to momentum space properties of Kallen-Lehmann spectral function; its conceptual pillar is rather the weakening of localization of charge-carrying fields as a result of de-localization through interactions with string-localized vectorpotentials (see next section). The infrared anomalous power cut in the K-L spectral function starting at the mass of the charged object as well as other momentum space anomalies are consequences and not the cause of the unexpected deviation from particle behavior (unexpected at least from the Lagrangian gauge theory viewpoint which does not reveal any eye-catching particularity). Whereas in  $d=1+1$  formal zero mass scalars play a prominent role, in  $d=1+3$  conformal scalars with anomalous dimensions cannot generate the noncompact localization which one needs to get the typical power cuts in the K-L two-point function; one really need string-like potentials appearing in the interaction density (next section)

Since most of the perturbative arguments in  $d=1+3$  about Maxwell charges inherit the mentioned problems of charged fields in the gauge theoretic setting, it is useful to know that there exists a rigorous conceptual argument based on a quantum field adaptation of the Gauss law [14]. The arguments can be found in Haag's book, it may however be helpful to remind the reader of its main content. One starts from a  $t - r$  smeared field strength

$$F^{\mu\nu}(f_R) = \int f_R(t, r) dt dr \int F^{\mu\nu}(t, r, \theta, \varphi) f_2(\theta, \varphi) d\theta d\varphi \quad (7)$$

$$f_R(t, r) = R^{-2} f\left(\frac{t}{R}, \frac{r}{R}\right)$$

The test function smearing of the singular pointlike fields is necessary in order to have a well-defined operator, for the classical field strength this would be superfluous. The angular integral defines an operator which represents the average flux through a sphere of radius  $r$  at time  $t$ . There are two physically motivated assumptions about the charged state of interest  $\omega$

$$\begin{aligned} \omega(F^{\mu\nu}(f_R)) &\neq 0 \\ \omega(F^{\mu\nu}(f_R)^2) &< \infty \end{aligned} \quad (8)$$

which is interpreted as a consequence of a quantum adaptation of Gauss's law i.e. the expectation value of the flux in a charged state deviates significantly from the vacuum, but the fluctuation should remain as bounded as they are

in the vacuum since the correlation of the field strength for large distances should not be influenced by the presence of a charge. Whereas the previous formulae specify the mathematical definition of electromagnetic flux through a spatial surface and charged state, the following commutation relation between the mass operator and the averaged field strength is at the heart of the matter:

$$[M^2, F^{\mu\nu}(f_R)] = iR^{-1}(P^\sigma F^{\mu\nu}(f_\sigma)_R) + F^{\mu\nu}((f_\sigma)_R)P^\sigma \quad (9)$$

It then follows that the state  $\omega$  cannot have sharp mass i.e. an electrically charged particles is necessarily an infraparticle. Furthermore the algebraic Lorentz symmetry is not unitarily implemented in a charged state (spontaneous symmetry breaking) and even stronger: the momentum direction of a asymptotically removed charged particle is a superselected quantity, since an infraparticle is inexorably burdened by infinitely many infrared photons which only can be pushed further into the infrared, but not eliminated.

That especially the last consequence appears strange to us is because our intuition about charged states has been formed by charged particle in QM where there is no superselection rule forbidding the coherent superposition of electrons with different momenta. But as in real life, we cannot accept the good parts of our most successful theory and reject structural consequences which we do not like. Of course part of this difficulty lies in the discrepancy between our structural knowledge and the present state of art about perturbation theory. The following sections are dedicated to the problem how to overcome these problems through a radical reformulation of gauge theory.

As mentioned on several occasions, interacting QFT is in many aspects much more radical than QM. Using a picturesque metaphor one may say that it realizes a benevolent form of Murphy's law: *states which are not interdicted by superselection rules to mutually couple, do inevitably couple*. The culprit for this complication (or, depending on one's viewpoint, this blessing) which contributes a fundamental aspect to QFT which is not shared by QM, is the inexorable occurrence of the kind of vacuum polarization resulting from the realization of the locality principle in the presence of an interaction; the sharing of  $\hbar$  does not bring them closer. So the question arose whether the charged one-particle states can remain unaffected. To exemplify what may happen, the idea of infraparticles was proposed in the mathematically controllable context of soluble two-dimensional models for which the anomalous power cut in the variable  $(\kappa - m)$  of the Kallen-Lehmann spectral function resembled the power behavior in the YFS work for the soft photon inclusive cross section.

In the 70s there appeared the first  $n^{th}$  order perturbative calculation which addressed the infrared properties of charge-carrying fields; they could be interpreted as confirming the momentum space infraparticle structure of charged states [15]. But this did not close the issue since they had two drawbacks; first they did not deal with gauge invariant charged fields and second they had nothing to say about the possible weaker localization, which in those 2-dimensional infraparticle models was the root of the infrared problem. Some of the perturbative observation were subsequently derived in a context which does not

directly refer to perturbation theory. For example the statement that there are no states with nontrivial charge nor states on which the Maxwell equations hold on indefinite metric spaces (as the Gupta-Bleuler or BRST settings) showed that the covariant perturbation theory involving charged fields has very serious physical defects [17][18]. At best one can use the pointlike field formalism for the calculation of gauge invariant vacuum expectation values and with the help of the Wightman reconstruction theorem arrive at a new Hilbert physical Hilbert space and physical (gauge-invariant) operators acting in it. Focussing attention on the local observables, the problem of infraparticles and spontaneous breaking of the Lorentz symmetry in charged states was taken up in [19] Further insights came from comparing the quantum problem of localization of charges with semiclassical arguments [20].

The most important structural (nonperturbative) enrichments, which came out of the infrared problem and its roots in noncompact localization, are certainly the aforementioned conclusions drawn from an appropriate formulation of the quantum Gauss law in conjunction with a nontrivial charge which led to charged states with an infinite extension. In the next section we will start to close the large gap between structural insight and computational implementation.

## 4 Localization peculiarities of zero mass Wigner representations

In this section we will start to address some recent theoretical observations.

There is a very subtle aspect of modular localization which one encounters in the second Wigner representation class of *massless finite helicity representations*<sup>17</sup> (the photon-graviton class) which recently attracted some attention [22]. Whereas in the massive case all spinorial fields  $\Psi^{(A,\dot{B})}$  the relation of the physical spin  $s$  with the two spinorial indices follows the naive angular momentum composition rules [21]

$$\left|A - \dot{B}\right| \leq s \leq \left|A + \dot{B}\right|, \quad m > 0 \quad (10)$$

$$s = \left|A - \dot{B}\right|, \quad m = 0 \quad (11)$$

the "covariantization" of the zero mass finite spin representations leads to a much stricter relation between the given physical helicity and the possible dotted/undotted components of the covariant spinorial formalism for which the Poincaré covariance can be extended to conformal invariance. For helicity  $s = 1$  the best one can do is to work with covariant field strength  $F_{\mu\nu}$  which in the spinorial formalism correspond to wave functions  $\Psi^{(1,0)}$ ,  $\Psi^{(0,1)}$ ; the desired vectorpotential representation  $\Psi^{(\frac{1}{2},\frac{1}{2})}$  corresponding to the classical vectorpotential

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<sup>17</sup>There are 3 positive energy classes: massive, massless with finite helicity and massless with infinite spin. The first and the third have the largest cardinality.

$A_\mu(x)$  simply does not occur in the Wigner representation theory based the use of Hilbert space <sup>18</sup>. Hilbert space positivity is of no concern in classical field theory; A pointlike classical field as  $A_\mu(x)$  is a classical field as any other constraint classical field i.e. as a member of an infinite dimensional subspace of a field space subject to the classical BRST and/or Batalin-Vikovitski formalism.

Most particle theorists who access QFT by quantizing classical field structures do not think much about why such a procedure leads inevitably to a loss of Hilbert space (Gupta-Bleuler, BRST formalism) which is the basic pillar of QT; they only take an indirect notice and often are not aware that they have left the quantum physical terrain during the calculations to which they must return at the end of the day in order to have physically interpretable results. The reason is that quantum phenomena which have no counterpart on the classical side are easily overlooked in a classical parallelism as quantization; but they stick out very clearly in an intrinsic approach to QFT as Wigner's representation theoretical setting.

At the root of this problem is a *fundamental clash* between the existence of pointlike covariant generating fields (or corresponding generating wave functions in the Wigner representation space) and the existence of spinorial fields which do not satisfy the restricted relation (11). Let us for brevity introduce the following terminology: all fields fulfilling this restricted relation will be called "field strength" and the ones which cause that clash are "tensorpotentials". In the case of  $s=1$  the field strength  $F_{\mu\nu}$  would be the field strength in Maxwell's theory whereas the quantum theoretically incriminated  $A_\mu(x)$  is the vectorpotential; for  $s=2$  the field strength is a 4-index tensor whose symmetry properties are those of the Riemann tensor etc. The setting can be generalized to half-integer  $s$  in which case  $s=3/2$  (Rarita-Schwinger field) is the lowest spin for which the clash of spinor potentials with the Hilbert space structure occurs. It will be seen below that the tensor representation of the representation type  $D(\frac{s}{2}, \frac{s}{2})$  are the most interesting ones.

The resolution of the clash between the existence of pointlike potentials and the Hilbert space of QT is very interesting. Instead of ceding on the side of the Hilbert space, the clear message is to keep the latter but do not insist in pointlike covariant localization but rather allow semiinfinite stringlike generators<sup>19</sup>. In order to not be misunderstood, all the  $(m=0, s \geq 1)$  Wigner representations possess pointlike generators but their covariantization does not produce those covariant fields which one needs for formulating renormalizable interactions.

As mentioned the standard path is to follow "quantum gauge theory" namely to keep pointlike fields and the well-studied perturbative formalism at the prize of an indefinite metric setting which usually comes with the introduction of additional unphysical degrees of freedom (ghosts). Since in typical perturbation

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<sup>18</sup>The relation between Wigner's one-particle representation theory and free fields will be explained in more detail below. In fact there is a one-to-one correspondence which permits to use the same letter for both (see below).

<sup>19</sup>As a pointlike field is a generator (the distribution-theoretical limit) of compact localized operator algebras, the string-like generator serves the same purpose for noncompact regions in case the pointlike generation is not possible.

calculations one does not use the Hilbert space norm for control of convergence (as Schwartz inequality) there is no problem. At the end of the calculation one has to reconvert the calculated correlation functions (e.g. by cohomological arguments) into a Hilbert space setting; this is the step from gauge variant to gauge invariant objects. To avoid any misunderstanding we are not proposing a new string localized Hilbert space based approach because of any suspicion that the gauge theoretical setting may be incorrect. Rather the reason is that the latter is incomplete because the physically most important charge carriers are not described within the standard gauge formalism; hence looking for an alternative has not only a philosophical side but there are also hard-core physical reasons. masks important physical properties for example the noncompact localization of charges, not to mention even more serious problems in case of Yang-Mills interactions (just those problems which are connected to confinement and invisibility of states). All these nonlocal (more precisely stringlike) effects must happen in the last step namely the cohomological descent; this radical change from pointlike gauge variant to stringlike gauge invariant overburdens the cohomological step apart from those objects which remain pointlike which are the pointlike gauge invariants. The most important physical objects as the charged fields remain outside the range of this gauge formalism.

As will be seen keeping the Hilbert space and not imposing impossible requirements on localization leads automatically to string-localized covariant potentials  $\Psi^{(A,\vec{B})}(x,e)$  which contain a spacelike string direction and are localized on  $x + \mathbb{R}_+ e$  for all  $(A, \vec{B})$  as in (10) which includes the covariant vectorpotential  $A_\mu(x,e)$  for  $s=1$  and the tensorpotential  $g_{\mu\nu}(x,e)$ . The field strength for spin  $s$  have scale dimension which  $s+1$  (the ones with higher dimensions can be written as derivatives) whereas the potentials have scale dimensions which fills the space between 1 and  $s+1$ , more precisely  $1 \leq \dim \Psi^{(A,\vec{B})}(x,e) \leq s$ . Of particular importance are the potentials  $\Psi^{(\frac{s}{2}, \frac{s}{2})}(x,e)$  because their scale dimension is  $\dim \Psi^{(\frac{s}{2}, \frac{s}{2})} = 1$  independent of  $s$ ; the previous fields  $A_\mu(x,e)$  and  $g_{\mu\nu}(x,e)$  are examples. They lead to interactions which are renormalizable in the sense of power counting. For pointlike fields renormalizability in the sense of power counting is synonymous with renormalizability whereas for string-localized fields this still must be established.

Since this clash between quantum theoretical positivity/unitarity and the existence of pointlike generators with a prescribed covariant transformation property is central to our new proposal, some more remarks are appropriate. The problem of constructing covariant free fields from Wigner's unitary representation theory of the Poincaré can be systematically solved in terms of intertwiners  $u(p,s)$  which are  $(2A+1) \binom{2\vec{B}+1}{2}$  component functions on the mass shell (which then may be rewritten into the tensor calculus). These intertwiners (between the unitary and the covariant representation) and their adjoints can be systematically computed, either by using group theoretic methods [21] or "modular localization" [22]. For the massless finite helicity case with its different "little group" most of the intertwiners do not exist (among them the vectorpotential); only those for which the spinorial indices are related to the

physical spin in the more restrictive manner as in the second line (10) remain available. All the missing covariant realizations can however be recovered if one allows the intertwiners to be dependent on a string direction  $u(p, e)$ . This construction is most conveniently done in the modular localization setting [22].

This construction has no counterpart in the classical Lagrangian setting and hence cannot be formulated in the functional integral setting (perhaps one reason why the string-localized potentials have only been noticed recently). But fortunately perturbation is intrinsically defined in QFT and does not need any crutches from the Lagrangian formalism; whether free fields are Euler-Lagrange fields or not is of no relevance for the working of perturbation theory [27]. However the importance of causal locality which is most visible in the Epstein-Glaser approach becomes even more important in passing from pointlike to stringlike fields.

The remaining question is then why does one need vectorpotentials if the field strength wave functions generate already the whole Wigner space or, if in interacting QED the quantum field strength together with the charged matter fields generate the full Hilbert space and form an irreducible set of operators in it? The short answer is that one does not need it for the description of QED but the string-localized potentials play a crucial role in understanding the delocalization of the charges; without having the potentials transfer their stringlike localization to the matter fields the nonlocality of the latter remains a mystery. But even staying in the free Maxwell theory the use of a pointlike vectorpotential together with Stokes law would lead to a zero result for the Aharonov-Bohm effect whereas the string-localized potentials leads to the correct effect.

Some of the answers can already be given in the free theory in terms of the generalized Aharonov-Bohm effect (next section). The implementation of renormalizable interactions adds additional reasons.

If one keeps the quantum (unitarity, Hilbert space, probabilities) aspects, then the way out is to relax the pointlike localization which underlies the Lagrangian quantization approach. Localization is important for the physical interpretation and the derivation of scattering theory, but the localization principle does not require to be realized in a pointlike generating manner, i.e. whereas there is no lee-way on the side of the quantum theory requirements, there is no principle in QFT which requires that a theory can be point-like generated. The mathematical question about the tightest localized generating wave function (technically: wave function-valued distribution) which by smearing with test functions generate the full Wigner representation space has been answered: all positive energy representations have semiinfinite stringlike distribution-valued generators and the massive as well as the massless finite helicity representations permit pointlike distribution-valued generating wave functions (generalized field strengths).

Only the zero mass *infinite* spin representation are intrinsically string-localized in the sense that there is no "field strength" which generates the same representation [26]. In [22] it was conjectured that also the associated QFT has no pointlike generated subalgebras based on the argument that their existence would lead to implausible consequences, but the issue is not completely settled

on the level of mathematical physics<sup>20</sup>. there is also the aforementioned curious difference between the two pointlike generated representation, which at closer inspection reveals a subtle distinction in localization aspects. In the massive case the little group is compact, whereas in the massless case it is the noncompact Euclidean stability group  $E(2)$  of a lightlike direction. In the finite helicity case this representation is finite dimensional, hence necessarily a unfaithful (degenerate) representation. Only in Wigner's infinite spin family this noncompact stability group ("little group") is faithfully represented.

All three families of representations massive, massless finite helicity and massless infinite spin are positive energy representations and there is a structural theorem [23], stating that all unitary positive energy representations of the Poincaré group (irreducible or not) can be generated by semiinfinite string-localized fields. But only in case of the infinite spin family (infinite dimensional representation of the stability group) this is the best possible localization. For the other two families the best (sharpest localized) generators are pointlike fields (operator valued distributions in the associated free field theory) which makes them accessible via the classical-quantum parallelism known as quantization.

The above observation about the existence of gaps in the spinorial covariance spectrum (10) means that even though both families of spinorial representations are finite dimensional, the noncompactness of the little group still makes its presence felt by not allowing most of the spinorial generators which occur in the massive case. Using a terminology which generalizes the ( $m=0$ ,  $s=1$ ) case of (noninteracting) electromagnetism, we may talk about pointlike "field strengths" and their associated string-like "potentials", which taken together *reconstitute the full spinorial spectrum* in the first line of (10). So from now on "field strengths" denote the covariant pointlike objects the second line of (10), whereas "potential" is the generic terminology for the string-localized remainder which recover the full spinorial spectrum of the first line.

The main idea is of course that, although string localized vectorpotentials do not fit into the standard formalism, it is better to face the problem on its physical side; always with the increased awareness that localization is the dominant principle, and in order to uphold it rather change the formalism than to compromise on physical principles. In order to avoid being misunderstood, we are not criticizing the gauge theory formalism in its efficiency to deal with local observables and we even have some sympathy for the temporary trespassing of the most cherished principles of QT by ghosts in the name of computational efficiency. Without the contributions of Veltman and t'Hooft, Faddeev and Popov and the BRST formalism of Stora et al., a consistent extension of the renormalization setting of QED to the standard model would not have been possible. But meanwhile almost 40 years have past, and although most people agree that the theory is nowhere near its closure, nothing of conceptual significance has happened. It is natural in such a situation to search for unexplored corners of QFT and the issue of localization, which is central to the present work, is

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<sup>20</sup>In particular since there has been a recent claim (without proof) to the contrary by Ch. Köhler, Institut fuer Theoretische Physik, University Goettingen, work in progress.

certainly a rather dark corner even in QED. The attempt to complete a theory may still turn into a unexpected radical change.

This paper is a plea to follow the localization principles and develop a new string-localization compatible formalism. We will present some of the first steps in this direction. It is worthwhile to mention already here, that the origin of the string-localized electric charge-carrying fields in QED, including their infrared aspects, is the result of the interaction of the matter current with the stringlike vectorpotentials. Whereas the influence of the stringlike localization on their own physical properties in selfinteracting Yang-Mills models is hard to foresee without understanding their infrared aspects, it is clear that the localization of the linearly related field strength in abelian gauge theories remain pointlike and that although the physical content of QED can be described in terms of physical string-localized matter fields  $\psi(x, e)$  and pointlike  $F_{\mu\nu}(x)$ , the stringlike vectorpotentials remain indispensable in the formulation of the interaction and the perturbative calculations.

Since such perturbative calculations are fairly involved and the cohomological descent from gauge-variant to gauge-invariant correlation is only simple for correlations involving  $F_{\mu\nu}(x)$  but not  $\psi(x, e)$ , it is not without interest that the quantum version of the Aharonov-Bohm like (next section)<sup>21</sup> permits to see the important role of the de-localized vectorpotential.

It is an ineradicable prejudice to believe that perturbation theory has to be set up in terms of quantized Lagrangians and functional integrals. The conceptually most pleasing perturbative approach consists in coupling covariant free fields (obtained as above by covariantizing the 3 classes of positive energy Wigner representations) together in form of invariant polynomial interactions<sup>22</sup> ("causal perturbation theory") which only contain pointlike fields and stringlike potentials of short distance scaling dimension  $d_{sca} = 1$ , since any higher dimensional fields/potentials would violate the power-counting requirement. Whether the coupled free fields obey an Euler-Lagrange equation is irrelevant for this perturbation theory, a fact which was already known to Weinberg [27].

It is another equally ineradicable misconception (usually related to the previous one), that the perturbative treatment of QFT contains intrinsic infinities which are "renormalized away". Fact is that certain implementations of perturbation theory, especially those which treat the problem in the quantum mechanical spirit of bringing certain operator functions of free field under control without paying much attention to the fact that fields, even in the absence of interactions, are rather singular objects which want their mathematical role as operator-valued distributions to be taken serious. If one does not do this, one still has a chance, but there is a prize to be paid: the removal of cutoff infinities

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<sup>21</sup>There is a curious analogy between the abandonment of the ether, which was mainly important for the post Maxwell-Lorentz development of particle physics, and the string-localization of potentials which only has severe consequences for the charged sector in the interacting theory.

<sup>22</sup>One may call it the interaction Lagrangian but the free fields/potentials used may and generally will not have a Euler-Lagrange structure. That perturbation theory does not require the existence of a Lagrangian for a free field was already known to Weinberg [27].



(or ad hoc regularization parameters). But there is always a finite way to arrive at those results, and if this would not be so, the whole renormalization approach would not have credibility.

For  $s \geq 1$  such potentials with  $d_{sca} = 1$  exist for all helicities. The property which is crucial in this approach is the localization structure, in fact the intertwiner formalism which leads to the spinorial fields (10) can be solely based on modular localization instead of group theory [22]. It is therefore not surprising that also the renormalization procedure can be built on the iterative fulfillment of the localization principle combined with a requirement of keeping the scaling degree of the counter-terms at its minimal possible value: the string-extended Epstein-Glaser approach.

For completeness and also for making some subsequent speculative remarks more comprehensible, it is important to say something about the large<sup>23</sup> third family of *infinite spin representations*. These are irreducible massless representations in which the euclidean  $E(2)$  stability subgroup is faithfully represented. In this case the representation theory of the little group leads to an infinite dimensional Hilbert space [22]. The Casimir invariant of the little group (i.e. the  $E(2)$  analog of the mass operator) takes on continuous values. In order to avoid the somewhat misleading terminology "continuous spin" in the older literature associated with a continuous representation of the "little" Hilbert space, it may be more appropriate to follow the recent terminology and refer to the "infinite spin representations".

Before we return to the discussion of consequences of stringlike localization, it is helpful to formalize the covariant fields for all three families. Some of these formulas can be found in the first volume of Weinberg's book [21] e.g. the following formula for massive free fields

$$\Psi^{(A,\dot{B})}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int (e^{-ipx} u^{(A,\dot{B})}(p) \cdot a(p) + e^{ipx} v^{(A,\dot{B})}(p) \cdot b^*(p)) \frac{d^3p}{2\omega} \quad (12)$$

where the dot stands for the sum over  $2s+1$  spin component values. The operators  $a^\#$  and  $b^\#$  are the momentum space annihilation/creation operators which transform according to the respective Wigner representation. The intertwiner  $u^{(A,\dot{B})}(p, s_3)$  and their charge conjugate counterpart  $v$  convert the Wigner representation into the covariant representation. They are rectangular matrices transforming a  $2s+1$  component Wigner spin into a  $(2A+1)(2\dot{B}+1)$  component covariant space. For a given physical spin  $s$  there is an infinity of possibilities of which one only uses the ones with low  $A, \dot{B}$  values which happen to have the lowest scaling degrees. The generating wave functions of the beginning of this section are obtained by replacing the Wigner creation/annihilation operators by the function  $f(p) = 1$ .

As mentioned, Weinberg's method to compute these intertwiners was group theoretical, but one can also base the computation on modular localization [22];

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<sup>23</sup>Although it is a zero mass representation, the faithfulness of the representation of the little group brings a continuous parameter into the game which leads to a higher cardinality of representations than in the finite spin case.

this is not surprising since covariance and locality of states are closely linked.

Practically the same formula, with the only change in the range of  $s_3 = \pm s$  and different expression for the intertwiners, holds for the massless case. However there is an important caveat, the formula exists only for the restricted  $A, \dot{B}$  values in the second line of (10) i.e. only for field strengths.

If one allows string-localization one can recover all the lost spinorial representations. These "potentials" are (by definition) all string-localized and obey for  $|A + \dot{B}| \geq s \geq |A - \dot{B}|$ , (field strengths excluded i.e.  $s \neq |A - \dot{B}|$ ) the following formula

$$\Psi^{(A, \dot{B})}(x; e) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int (e^{-ipx} u^{(A, \dot{B})}(p, e) \cdot a(p) + e^{ipx} v^{(A, \dot{B})}(p, e) b^*(p)) \frac{d^3 p}{2\omega} \quad (13)$$

$$A^\mu(x, e) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int (e^{-ipx} u^\mu(p, e) \cdot a(p) + e^{ipx} \overline{u^\mu(p, s_3; e)} \cdot a^*(p)) \frac{d^3 p}{2\omega} \quad (14)$$

where the dots stand for the sum over the two helicities  $s_3 = \pm s$  and the vectorpotential intertwiner, which has been written down separately, has the form

$$u^\mu(p, s_3; e)_\pm = \frac{i}{pe + i\varepsilon} \{(\hat{e}_\mp(p)e)p^\mu - (pe)\hat{e}_\mp^\mu(p)\} \quad (15)$$

and the  $\hat{e}_\pm$  denotes the two photon polarization vectors to be distinguished from the string direction. These operators transform covariantly and have stringlike commutation relations

$$U(\Lambda)\Psi^{(A, \dot{B})}(x, e)U^*(\Lambda) = D^{(A, \dot{B})}(\Lambda^{-1})\Psi^{(A, \dot{B})}(\Lambda x, \Lambda e) \quad (16)$$

$$\left[ \Psi^{(A, \dot{B})}(x, e), \Psi^{(A', \dot{B}')} (x', e') \right]_\pm = 0, \quad x + \mathbb{R}_+ e > x' + \mathbb{R}_+ e'$$

As expected, the scaling degree of the potential is  $d_{sca}(A^\mu(x, e)) = 1$  i.e. better than that of the field strength. The resulting two-point function is of the form [25]

$$\langle A_\mu(x; e) A_\nu(x'; e') \rangle = \int e^{-ip(x-x')} W_{\mu\nu}(p; e, e') \frac{d^3 p}{2p_0} \quad (17)$$

$$W_{\mu\nu}(p; e, e') = -g_{\mu\nu} - \frac{p_\mu p_\nu}{(p \cdot e - i\varepsilon)(p \cdot e' + i\varepsilon)} (e \cdot e') +$$

$$+ \frac{p_\mu e_\nu}{(e \cdot p - i\varepsilon)} + \frac{p_\nu e'_\mu}{(e' \cdot p + i\varepsilon)}$$

The presence of the last 3 terms is crucial for the Hilbert space structure; without them one would fall back to the indefinite metric and negative probabilities.

Either from the two-point function or more directly from the form of the intertwiners one reads off the following two relations:

$$\partial_\mu A^\mu(x, e) = 0 = e_\mu A^\mu(x, e) \quad (18)$$

These formulas are not imposed but are consequences of the requirement of having a covariant vectorpotential in the Wigner Hilbert space which, since it cannot be point-local, brings in an additional geometric parameter  $e$ .

Note that the string-localized potential looks like the axial gauge potential in the gauge theoretical setting, where  $e$  is a gauge parameter which, different from the above definition, is inert against Lorentz transformations.

The difference between the gauge interpretation and string-localization is more conceptual than formal. The fluctuation in both parameters  $x, e$  ( $e$  a point in 3-dim. de Sitter space) is very important for the lowering of the  $x$ -dimension, namely  $d_x = 1$ , at the cost of fluctuation in  $e$  which manifest themselves as string-caused infrared divergences; the latter explain why the axial gauge, which overlooks the fluctuations in  $e$  is intractable (it was never of any computational use); despite of its welcome Hilbert space structure, the fluctuations in  $e$  prevented to control these divergencies, in fact it was not even possible to understand their physical origin. The localization based approach on the other hand treats the string potentials as distributions in  $x$  and  $e$  and addresses the question if and how the different  $e_i$  have to coalesce at the end. The axial gauge is besides the Coulomb gauge the only one which can be accommodated in a physical Hilbert space.

The interpretation of the  $e$  as a fluctuating covariant string direction rather than as fixed gauge parameter in the interpretation as a string-localized potential is the only meaningful interpretation. Hence the standard argument in favor of gauge invariance based on returning to a physical space from the unphysical indefinite metric formulation has lost its conceptual basis. But a new problem has emerged, namely how to treat fluctuating string directions. If one has grown up with a "gauge principle", this may seem surprising, but the surprise should not be new, one could have asked the crucial question "does the axial gauge with its infrared divergencies (even for off-shell expectations of matter fields) fit into the standard gauge-ideology?" already a long time ago. The answer to this question is that it does not, it is really a Hilbert space description in which the pointlike was changed with semiinfinite stringlike localization; but it required the use of the modular localization concept [23][22] to raise the awareness about this issue.

In order to obtain a theory in which the interaction between the vectorpotentials and matter leads to a subalgebra of local observables, one needs a relation which connects the potential for two different directions

$$\begin{aligned} A^\mu(x, e) &\rightarrow A^\mu(x, e') + \partial^\mu \Phi(x; e, e') \\ \Phi(e, e'; x) &= \int e_\mu A^\mu(x + te', e) dt \end{aligned} \quad (19)$$

The proof of pointlike locality of certain fields amounts to the  $e$ -independence; this is not different from the proof of independence of gauge parameters in the standard gauge theoretical setting. But the main purpose of this formalism is not the identification of local observables and the calculation of their correlation functions but rather to incorporate the string-localized charged fields and their infraparticles into the perturbative formalism.

In fact the string localization suggests to view the change of  $e$  as the result of two subsequent Poincaré transformations a Lorentz rotation  $\Lambda(e, e')$  around the origin which transforms  $e$  into  $e'$  and a conjugation by a translation, hence together

$$A(x, e') = V(e, e'; x)A(x, e)V(e, e'; x)^*, \quad V(e, e'; x) \equiv U(x)U(\Lambda(e, e'))U(-x) \quad (20)$$

$$V(e, e'; x)\varphi^*(x)\partial_\mu\varphi(x)V(e, e'; x)^* - \varphi^*(x)\partial_\mu\varphi(x) = \partial_\mu\Phi(e, e'; x)$$

which leads to an interpretation of  $\Phi$  in terms of spacetime operations. This in turn suggests that in a theory in which the stringlike vectorpotential interacts with matter fields as in QED, *composite charge neutral operators* which contain derivatives acting between charge-carrying operators as  $\varphi^*\partial_\mu\varphi$  should "feel" the  $e \rightarrow e'$  change by producing additive  $\partial_\mu\Phi$  terms as in the second line above, so that gauge invariance formally corresponds to string independence in the new setting. In view of the fact electrically charged fields cannot be better than string-localized and that the Lorentz invariance is spontaneously broken in charged sectors one expects that an operational description of the  $e$  change may connect these two facts within a perturbative setting. In that case one would have three kinds of behavior under change of  $e$ :  $e$ -independent i.e. pointlike localized operators corresponding to gauge invariant observables, operators which change additively under changes of  $e$  similar to a change under a symmetry transformation and operators on which these symmetry-like transformations are spontaneously broken. This description, if possible, would strengthen the old observations of an interrelation of gauge and spacetime properties (Lorentz-covariance and localization) and could reconcile rigorous structural observations as the spontaneous breaking of the Lorentz symmetry in electrically charged sector of the theory with the formalism of perturbation theory for string-localized objects. In case of selfinteracting vector- or tensor-potentials, as in Yang Mills and Einstein-Hilbert interactions, the changes under  $e$  in the interacting theory are not the same as for free fields. All these problems, which are connected with string-localization will be left to future work.

A closely related question is whether Hertz-potentials, which were introduced by Hertz into Maxwell's theory as useful computational tools, have also a beneficial role to play in QED. Formally they would be described by string-localized antisymmetric tensor fields with zero scale dimension so that derivatives and exponentials have positive dimensions. They appear in the work of Penrose in connection with asymptotic behavior of zero mass higher spin fields [24].

Another special case of significant interest is the case of  $s=2$  whose field strength with the lowest scale dimension is an object  $R_{\mu\nu\kappa\lambda}$  with the linear

properties of the Riemann tensor and an  $d_{sca} = 3$  and a string-like potential  $g_{\mu\nu}(x, e)$

$$\langle g_{\mu\nu}(x, e) g_{\kappa\lambda}(x', e') \rangle = \int \frac{d^3 p}{2p_0} e^{-ip(x-x')} W_{\mu\nu\kappa\lambda}(p; e, e') \quad (21)$$

$$W_{\mu\nu\kappa\lambda}(p; e, e') = W_{\mu\alpha\nu\beta\kappa\rho\lambda\sigma}^R \frac{e^\alpha e^\beta e'^\rho e'^\sigma}{(e \cdot p - i\varepsilon)^2 (e' \cdot p + i\varepsilon)^2}$$

where the superscript  $R$  refers to the field strength 2-point function whose 8 tensor indices correspond to the 4 tensor indices of the independent field strength and reflect the fact that the relation between the potential and the field strength is the linearized version of that between the metric tensor and the Riemann tensor [25]. The  $e$ -dependent factor obviously improves the short distance properties. As expected the  $g_{\mu\nu}$ -potential has  $d_{sca} = 1$ .

String-localized potentials with  $d_{sca} = 1$  can also be constructed for massive theories, even though there is no compelling reason from the viewpoint of the Wigner representation theory for doing this since pointlike fields with a fixed physical spin exist for all spinorial pairs fulfilling (10).

In this case the *only reason would be the power counting requirement*. Since the increase of the short distance dimension with spin happens independent of the presence of a mass, there would be no renormalizable interaction of a spin one massive  $A_\mu(x)$  field with other  $s=0$  or  $s=\frac{1}{2}$  matter fields, the only way out is to take a string-localized massive  $A_\mu(x, e)$  with  $d_{sca} = 1$  instead of  $d_{sca} = 2$  for  $A_\mu(x)$ . This enlarges the number of candidates for renormalizable interactions from a finite number to infinitely many. But even if some interactions which are power counting renormalizable turn out to lead to mathematical consistent theories, unless they have local observables in the form of pointlike generated subtheories, they are physically unattractive.

For infinite spin one finds [22][25]

$$\Psi(x; e) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int (e^{-ipx} u(p; e) \circ a(p) + e^{ipx} \sum_{s_3=\pm s} v(p; e) \circ b^*(p; e)) \frac{d^3 p}{2\omega}$$

$$u(p; e)(\kappa) = \int_{\mathbb{R}^2} d^2 z e^{ikz} (\xi(z) B_p^{-1} e)^{-1+i\alpha} \quad (22)$$

where  $B_p$  is the  $p$ -dependent family of Lorentz-transformation, selected in such a way that they transform the reference  $\bar{p}$  on the irreducible orbit into the generic  $p$ , and the circle product stands for the inner product in the "little" Hilbert space which consists of square integrable functions of a two-dimensional Euclidean space  $\bar{f} \circ g = \int \bar{f}(\kappa) g(\kappa) d^2 \kappa$ . So the Wigner  $a(p)^\#$  operators and the intertwiners depend on the euclidean  $\kappa$  variables make the dependence on  $e$  much more involved than in the finite helicity case. Nevertheless they are string-localized for all values of  $\alpha$ . The  $\kappa$  dependence of the intertwiners results in a stronger form of string localization than for zero mass potentials.

This stronger form of delocalization shows up in the fact that there are no pointlike field strengths [26]. In fact the operator algebra generated by these

fields apparently has no compactly generated observable (pointlike generated) subalgebras<sup>24</sup>. In general stringlike generators of algebras which cannot be assembled as a union from compact parts (as our favorite example of operators carrying a nontrivial electric charge) do generate reducible states under the action of the Poincaré group if applied to the vacuum which are pointlike generated; the localization of operator algebras and the localization of states are two different pairs of shoes. But representations of the Poincaré group which have infinite spin components in their reduction can only appear in operator algebraic structures which were string-localized.

The only states which are intrinsically string-localized are those associated to the infinite spin representation. Theories in which they occur would have serious problems with being accessible to observations<sup>25</sup>. This is because "counters" in QFT are compactly localizable or, in order to avoid vacuum polarization problems [28], they should be at least localizable in the sense of quasilocality. Such a counter cannot register an intrinsically string-localized state, so that quantum matter related to the third Wigner class remains "invisible" despite the fact that it carries nonvanishing energy-momentum (and hence susceptible to gravity). Theories containing such representations are candidates for "invisible" quantum matter. So maybe Weinberg's "no" at the time of writing his book should be weakened to "not yet".

## 5 Aharonov-Bohm effect for and violation of Haag duality

Suppose one generates the ( $m = 0, s \geq 1$ ) Wigner representation space (or the associated net of local algebras in the Wigner-Fock space) with pointlike field strength wave functions. Does the theory let us know that we forgot that there are string-localized potential which want to play an important physical role? In this case the Wigner representation should signal by its localization properties that there is a difference in localization between the massive to the massless representation, but does it really do this? Is there an intrinsic difference within the Wigner representation with respect to the modular localization structure which goes beyond the covariantization formula (10)?

There is indeed a subtle representation theoretical distinction which is connected with *Haag duality*. Whereas for simply connected convex double cone regions the localization spaces (real subspaces of the Wigner representation space, which are defined in terms of modular localization<sup>26</sup> (28)) one finds Haag duality

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<sup>24</sup>The pedestrian argument for bilinear operators in [22] can be generalized to monomials of arbitrary orders, but an elegant proof based on modular methods is still missing.

<sup>25</sup>This may have been the reason why Weinberg [21] dismissed them as unphysical, despite their fulfillment of the positive energy requirement.

<sup>26</sup>The K-spaces are real subspaces of the complex Wigner space which are defined as eigenspaces of the involutive Tomita S-operator. For a presentation of these spaces which is close to the spirit of the present paper we refer to [29].

for both the massive as well as the massless finite spin case

$$K(\mathcal{O}') = K(\mathcal{O})' \text{ or } K(\mathcal{O}) = K(\mathcal{O}')' \quad (23)$$

$$\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}')', \quad \mathcal{O} \text{ double cone} \quad (24)$$

but there are interesting differences for *non simply connected regions* as relative causal complements of a smaller spacetime double cone within a larger one (the causal completion of a torus) where (23) refers to the Wigner representation theory and (24) is the second quantized algebraic version. The interpretation of Haag duality in the standard (von Neumann) quantum theoretical setting of measurements is that any measurement which is compatible with all measurement inside a causally complete spacetime region, must be associated with the causally disjoint observables (24); this rule can only be broken for very special and important reasons.

One knows from some old (but unfortunately unpublished) work [30] that for free QED, i.e. for the (m=0,s=1) Wigner representation, the Haag duality breaks down for a spacetime region  $\mathcal{T}$  which results from the causal complement of a double cone inside a larger double cone or by sweeping an  $x - t$  two-dimensional double cone subtended from a spatial interval  $x \in [a, b]$ ,  $0 < a < b$ , by higher dimensional rotation around the origin. In d=1+2 the result would be topologically equivalent to the inside of a torus, whereas in d=1+3 it is the doubly connected 4-dimensional analog namely the causal completion of a 3-dimensional torus region at a fixed time. One then finds [30]

$$K(\mathcal{T}') \subsetneq K(\mathcal{T})' \quad (25)$$

$$\text{or } K(\mathcal{T}) \subsetneq K(\mathcal{T}')', \quad \mathcal{A}(\mathcal{T}) \subsetneq \mathcal{A}(\mathcal{T}')' \quad (26)$$

where in the second line we also wrote the violation of Haag duality relation in the interaction-free algebraic setting which are functorially related to the one formulated in terms of modular localized real particle subspaces  $K$  of the complex Wigner representation spaces.

These proper containment relations in the special case of the free electromagnetic field are quantum field theoretic analogs of the quasiclassical Aharonov-Bohm (also Ehrenberg-Siday) effect. This is an effect of a localized classical magnetic flux in an infinitely long solenoid exerts on a quantum mechanical electron which scatters on the solenoid even though it stays outside its thin magnetic flux tube. Its quantum field theoretic counterpart (A-B effect in QFT<sup>27</sup>) is more stringent; it states that despite the continued validity of Stokes theorem for the quantum magnetic field, the quantum magnetic potential has modular localization properties which differ from the classical intuition since classically it should be localized on (or at least near) the Stokes boundary circumference. In other words the effect would disappear if the magnetic potential would be a pointlike field as in gauge theory.

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<sup>27</sup>Usually the A-B effect refers to the semiclassical scattering of charged particles on a thin solenoid whose magnetic field remains inside.

This does of course not mean that gauge theory is misleading, it is only a warning against its unconstrained use. Matters of localization should only be discussed in a Hilbert space i.e. *after* having implemented the invariance under BRST transformations. But since the BRST "symmetry" acts in a nonlinear way, the construction of BRST invariant local correlations is very difficult and for this reason rarely done; it is especially the case if nontrivial Maxwell or Yang Mills charges lead to stringlike localizations after having done the cohomological BRST descent.

There are two remedies for the A-B effect at hand, either one introduces the notion of quantum cohomology of field strength [30], or one uses the *string-localized quantum vectorpotentials*. For free electromagnetic field the first choice is completely adequate, but in the presence of interaction only the formulation based on string-localized potentials has a good chance to permit the perturbative construction of physical charged fields and to really explain the origin of their de-localized nature.

In the cited work [30] the connection with the A-B effect was not mentioned and the representation theoretical basis of modular localization via intersection of wedges was not yet available. The calculation was done in the covariant field strength formalism of  $\vec{E}, \vec{H}$ . The main purpose was to formulate a warning against the use of pointlike vectorpotentials in QFT which leads to a contradiction with the A-B effect and its extension i.e. the violation of Haag duality (25). This, and the remark that string-localized magnetic potentials avoid this contradiction, is the precise reason why we revisited this age old problem. Since in the massive case the equality sign in (25) continues to hold, the toroidal Haag duality violation is the looked-for *intrinsic representation theoretic distinction* between massive and finite helicity massless representations.

Perhaps the best way to present this result is to say that the pointlike localization for massless vectorpotentials clashes with the Hilbert space positivity and since the latter is the essence of quantum theory, it is the pointlike localization which has to cede. The only generalization of pointlike localization turns out to be semiinfinite stringlike; the positive energy condition of unitary representations of the Poincare group does not require to introduce *generating* wave functions (or free fields) which are weaker localized than a semiinfinite string.

Following LTR one looks at a situation of two spatially separated, but interlocking regions  $\mathcal{T}_1$  and  $\mathcal{T}_2$  in which one represents as the smoothened boundary of two orthogonal unit discs  $D_1$  and  $D_2$  which intersect in such a way that the boundary of one passes through the center of the other. The delta function fluxes through the  $D_i$  are smoothened by convoluting  $\star$  with a smooth function  $\rho_i(\mathbf{x})$  supported in an  $\varepsilon$ -ball  $B_\varepsilon$ ; the interlocking  $\mathcal{T}_i$  are then simply obtained as  $\mathcal{T}_i = \partial D_i + B_\varepsilon$   $i = 1, 2$ . One computes the following objects

$$\begin{aligned} \text{Im}(e(\vec{g}_1), h(\vec{g}_2)) &\simeq [\vec{E}(\vec{g}_1)\vec{H}(\vec{g}_2)] = \int \vec{g}_1(x) \text{rot} \vec{g}_2(x) d^3x = \\ &= \int \rho_1(x) d^3x \int \rho_2(y) d^3y, \quad \vec{g}_i = \vec{\Phi}_i \star \rho, \quad \vec{\Phi}_i(\vec{f}) = \int_{D_i} \vec{f} d\vec{D}_i \end{aligned} \quad (27)$$

where we have written the result in two different ways, on the right hand side



the algebraic (commutator) expression and on the left hand side in terms of the associated Wigner wavefunction. The  $\Phi_i$  is the functional which describe the flux through  $D_i$ , a kind of surface delta function.

The calculation of wavefunction inner product and its associated symplectic form defined by its Imaginary part is more lengthy but straightforward. It is needed because it contains the information of the modular localization. We will omit it here and simply state the result. It confirms relation (25) since  $e(\vec{g}_1) \in K(\mathcal{T}'_1)$ ,  $h(\vec{g}_2) \in K(\mathcal{T}'_2)$  but none of these two wave functions are in the smaller spaces  $K(\mathcal{T}_i)$  since the algebraic right hand side (27) is definitely nonvanishing. The QFT A-B effect is the only known violation of Haag duality for which the duality violating operators cannot be used for a "Haag dualization" i.e. an extension process by which Haag duality can be recovered for the extended not of local algebras.

The calculation can be done entirely in terms of field strengths, there is no need to use potentials and their two point functions (17). The first term in (17) which contains only  $g_{\mu\nu}$  and no string dependence  $e$  would lead to an indefinite inner product if taken for the two-point function of vector potentials; in fact this would describe the indefinite two-point function of pointlike vector potentials in the covariant Feynman gauge; but restricted to the field strength it is perfectly positive. On the other hand the full two-point function (17) is positive, this was the main achievement of reconciling modular localization and positivity via string-localization. For the cohomological argument supporting the QFT A-B effect or breakdown of Haag duality, one does not need the potentials. It is only if one wants to have a more operational argument for the discrepancy between Stokes theorem and modular localization than that based on cohomology that one needs the free stringlike potentials.

However the operational formulation in terms of *string-localized potential become absolutely crucial in the presence of interactions* for the understanding of the properties of physical charges. I know of no cohomological argument in terms of which one can understand the localization properties of interacting Maxwell charges. The fact that the A-B effect disappears if one uses the pointlike (and hence non Hilbert space) magnetic potential  $\vec{A}(x)$  in Stokes theorem shows that one has to be very careful in drawing physical conclusions from the standard gauge formalism. Only after the imposition of BRST invariance and the cohomological descent (too difficult for correlation of charged field, only stated but never performed) has one left the slippery ground.

There is no all-clear in the context of interacting gauge theories. It is well-known that there is no electric charge in a formulation QED in which a pointlike vectorpotential is still present [17][19]. In such a case there exists only the nontrivial kinematical Dirac charge, but the Maxwell charge vanishes. As in the case of the above Aharonov-Bohm effect, this unphysical aspect disappears upon using the string-localized vector potentials in the Hilbert space formulation. Although we will not go into higher spin problems it may be interesting to remark on the side that the string-localization in Hilbert space has extensions to higher spin potentials whose scale dimension does not increase with  $s$  (with renormalizable interactions in the sense of power counting) whereas an extension

of the gauge formulation beyond  $s=1$  is not known.

The violation of Haag duality for conformal QFT on multifold connected spacetime regions is part of modular theory and this raises the question whether one can compute the modular group. The answer is positive and quite interesting; it will be deferred to an appendix.

Some more remarks about (the algebraic) Haag duality and its breaking are in order. Its validity for simply connected regions  $\mathcal{O}$  (24) defines a "perfect" world in which the quantum counterpart of the classical Cauchy propagation holds i.e. a local algebra is equal to that of its causal completion and the commutant of the algebra localized in the causal disjoint  $\mathcal{O}'$  is equal to the original algebra.

As mentioned, interesting situations arise when the world of local quantum physics is not perfect and the Haag duality is violated i.e. the right hand algebra is genuinely bigger than the left hand side. The most common violation results from an observable algebra which is localized in several disconnected spacelike separated double cones (separated intervals in the chiral conformal case [32]). In case the observable algebra possess localizable superselected charges, the right hand side for such a multi-disconnected region  $\mathcal{A}(\mathcal{O}')'$  is genuinely bigger because the *charge transporters* which carry the charge from one to the other region are in  $\mathcal{A}(\mathcal{O}')'$  but not in  $\mathcal{A}(\mathcal{O})$ ; the charge transporters are globally neutral, but they change the localization of charges between the localization regions.

Such models fall into the range of the DHR superselection theory [28]. The final result of this theory is the (unique) existence if a "field algebra", which contains all superselected charges and a compact symmetry group<sup>28</sup>, which acts on the field algebra in such a way that the observable algebra re-emerges as the fixed point subalgebra. In the chiral case one can even compute geometric modular groups for such situations. They are associated with higher diffeomorphism groups beyond the Moebius group [33] and they require to trade the standard vacuum with the so-called split vacuum. In all those cases the violation of Haag-duality is an indicator of the presence of charge superselection sectors and a global symmetry. In an appendix the reader finds an interesting illustration of such a situation.

The idea underlying the relation between the charge neutral observables and the charge carrying fields can be best by borrowing a famous phrase from Marc Kac in connection with Hermann Weyl's inversion problem namely "how to hear the shape of a drum?" If one substitutes drum by the full-fledged QFT containing globally charged fields and the perceived sound by the observable charge neutral fields, the existence of the superselection theory stands for the reconstruction of the full theory (containing all charges) from the observable "shadow". The existence and uniqueness of this inverse problem has given a significant insight into the inner workings of QFT; in particular it demystified the origin of inner symmetries, a concept which started in the 30s with Heisen-

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<sup>28</sup>The appearance of compact group theory via the localization properties of observable algebras is perhaps the most surprising aspect of the power of quantum localization [34].

berg's isospin. Morally something like this should also hold for local (Maxwell) charges, if one allows for the possibility that certain charges cannot be registered and probably not even produced in collisions of neutral particles (see section 7). However investigations in this direction did not lead to anything tangible. We view our approach as yet another attempt in this direction.

The violation of the Haag duality for the above doubly connected  $\mathcal{T}$  has a quite different physical message. First it can be detected already in the Wigner setting, so it has nothing to do with superselection sectors and vacuum polarization. To appreciate its message, it is indicative to imagine that the field strength can be derived from a pointlike potential  $A_\mu(x)$  which is the standard starting point of the indefinite metric gauge setting. In that case the fluxes will be supported inside the spacelike separated  $\mathcal{T}$ . The vanishing of the resulting expression is in flagrant violation of the above calculations. This shows that the standard indefinite metric gauge formalism is unreliable. As we pointed out before it is not wrong, but one has to carefully distinguish situations where it can be applied from those where it leads to incorrect conclusions<sup>29</sup>. For the present purpose it is the *strongest support for the introduction of stringlike potentials* in the absence of interactions which would not create any contradiction in the above calculation.

Both, the charge superselection problem and the problem of multi-connectedness are intimately related to the way in which models of local quantum physics realize the localization principle. The Aharonov Bohm effect is perhaps the most direct and simple illustration since it does not require composite fields and vacuum polarization.

To generalize this subtle violation of Haag duality to arbitrary  $(m = 0, s > 1)$  representations one needs a more adequate modular setting than the pointlike covariant field strength formalism used in [30]; the latter becomes increasingly complicated with the increasing number of tensor/spinor indices. A structural method which avoids the use of covariant field coordinatizations and which is a wave function preform of the net of localized algebras in the LQP formulation of QFT consists in the use of the Wigner representation theory of the Poincaré group. In order to loose the reader, I will try to use one paragraph to present at least some of the physical content of that impressive theory; for its mathematical backup see [31]

It has been known for a long time that the algebraic structure underlying free fields allows a functorial interpretation in which operator subalgebras of the global algebra  $B(H)$  are the functorial images of subspaces of the Wigner wave function spaces ("second quantization"<sup>30</sup>), in particular the spacetime localized algebras are the images of localized subspaces. Since localized subal-

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<sup>29</sup>The point here is that in gauge theoretic calculations one computes numbers and is normally one is not interested in localization. On the other hand the commitment to gauge invariant results is mainly a lip-service, physical correlations are easily characterized in terms of BRST invariance, but the computation of charge neutral composites is quite a different story.

<sup>30</sup>Not to be confused with quantization; to quote a famous saying by Ed Nelson: "quantization is an art, but second quantization is a functor".

gebras in QFT  $\mathcal{A}(\mathcal{O})$  are known to act cyclic and separating on the vacuum (the Reeh-Schlieder property), the conditions for the validity of the Tomita-Takesaki modular theory are fulfilled.

The start for constructing such subspaces  $K$  is always the localization in wedge regions  $W$ , since in that case the modular objects associated with the Tomita  $S$ -operator  $S = J\Delta^{\frac{1}{2}}$  are part of Wigner's representation theory, namely the antiunitary  $J$  and modular unitary  $\Delta^{it}$  coalesce with the spacetime reflection on the edge of the wedge and with the wedge-preserving boost respectively [28]. The real localization subspace  $K(W)$  is just the closed  $+1$  eigenspace of  $S$  and the associated dense complex standard space is its complexification

$$\begin{aligned} K(W) &= \overline{\{\varphi | S\varphi = \varphi\}} \\ K(W) + iK(W) &\text{ is standard} \end{aligned} \tag{28}$$

where standardness ( $K$  and  $iK$  have trivial intersection and taken together form a dense set) of spaces and operator algebras (intimately related to the Reeh-Schlieder property [28]) is one of the most important concepts of modular localization theory.

The modular objects for subwedge regions are determined by representing them in terms of intersections of wedges and showing the standardness of the associated subspaces. This is fairly easy for noncompact regions as spacelike cones  $\mathcal{C}$ , whose core is a semiinfinite spacelike string with an arbitrary small opening angle. Evidently they can be represented by intersections of wedges with a shared origin which then becomes the apex of the spacelike cone  $\mathcal{C}$ . In this case the standardness follows from the energy positivity [23] i.e. it is shared by all three Wigner representation. All 3 families have stringlike wave-function valued generators  $\Psi(x, e)$ , but only in the infinite spin case there are no better localized generators.

A systematic investigations along these purely modular lines would start with showing the standardness of compact double cone  $\mathcal{D}$  localized subspaces  $K(\mathcal{D})$  avoiding the use of covariant wavefunctions/fields and relying entirely on the Wigner representation theory. The double cones are the causally closed regions in terms of which the setting of algebraic QFT (spacetime-indexed nets) is defined. In that case the origin of the wedges cannot be fixed (to the origin of the Minkowski spacetime coordinatization) but have to be passed around a circle on the two-dimensional spatial boundary if a symmetrically chosen (around the coordinate origin) double cone. From concrete calculations with pointlike generating fields we know that generating wave functions  $\Psi^{(A, \dot{B})}(x)$  obtained from covariantization of the Wigner wave function do generate standard compactly localized subspaces. But only an abstract version of this proof will reveal the importance of the nature of the little group and its impact on the localization problem, i.e. why unitary representations of compact- and finite dimensional representations of noncompact little groups are compactly localizable, whereas the infinite spin representation is not ( $K(\mathcal{D}) = 0$ ).

The modular localization of states is much weaker than that of local algebras. Whereas for positive energy states on which the Poincaré group is

unitarily represented the modular localization leads can be expressed in terms of an antilinear operator which for wedge localization is of the form  $S = J_0 \Delta$

Having prepared oneself in this way, there remains the structural understanding of the generalized Aharonov-Bohm effect namely for multiply connected regions there exist observables which, although they commute with the field strength in the causal disjoint, are not expressible in terms of field strength inside the multiply connected spacetime region. Since the field strength determine the global properties, the generalized A-B effect is another manifestation of the holistic aspects of QFT in this case one which distinguishes massive from massless representation.

Although the QFT A-B effect has only been established for  $(m=0, s=1)$ , the role of the little group in localization leaves little doubt that there exists a generalization for  $s>1$ . Invoking a metaphoric principle namely that nature may have only few principles but an enormous variety of different manifestations, one is inclined to speculate that the increasing number of potentials with increasing  $s$  is associated with higher than double connectivity generalizations of the QFT A-B effect (and not only with an increase of the number of A-B operators in a geometric situation of  $n$  separated  $\mathcal{T}$  regions. This would shed light into the dark corners of higher spin quantum matter which has been closed to gauge theory inspired ideas. Finally there is the question of the action of the modular group in massless theories, especially in cases where one expects this to be geometric, as in the case of  $\mathcal{T}$ . The answer to any of these questions would require more mathematics and would lead too far away from the spirits of this paper for which this section only serves to illustrate that there are indication for the role of string-localized potentials already in the free field theories. But the importance of this unexplored suggests to return to it in a more specific future context.

The zero mass higher spin field strengths exhibits the above increase of scale dimension of pointlike generated field strength with spin and therefore shows a worsening of field strength associated short distance singularities. The bosonic potentials, namely  $n=s$  string-localized generators with increasing short distance dimensions fill the gap between  $d_{sca}=1$  up to  $d=s$  where  $s+1$  is the  $d_{sca}$  of the lowest dimensional field strength (the lowest dimension consistent with the second line in (10)). We already emphasized that the localization in a indefinite metric setting has no relation to the physical localization; this is the main message of the A-B effect and the violation of Haag duality for QFT.

The same second line (10) contains the considerably reduced number of spinorial descriptions for zero mass and finite helicity, although in both cases the number of pointlike generators which are linear in the Wigner creation and annihilation operators [22].

By using the recourse of string-localized generators  $\Psi^{(A,\dot{B})}(x,e)$  one can *restore the full spinorial spectrum* for a given  $s$ , i. e. one can move from the second line to the first line in (10) by relaxing the localization. Even in the massive situation where pointlike generators exist but have short distance singularities which increase with spin. there may be good reasons (lowering of

short distance dimension down to  $d_{sca}=1$ ) to use string-like generators. In all cases these generators are covariant and "string-local"

The explicit verification of stringlike locality is cumbersome because there are no simple x-space formulae for stringlike Pauli-Jordan functions. It is easier to avoid manifest x-space localization formulas and work instead with intrinsically defined modular localization subspaces. In fact the construction of the singular stringlike generators are not based on any gauge theory argument but rather a consequence of the availability of stringlike intertwiners for all unitary positive energy representations of the Poincare group. In the present setting the constraining equations (18) have nothing to do with a gauge condition but are rather a consequence of constructing intertwiners which localize on a string  $x + \mathbb{R}_+ e$  which is the next best possibility in cases where the compact localized subspaces are empty and their pointlike generators nonexistent. The pointlike aspect of the gauge formalism is only physically relevant in case of gauge invariant operators i.e. the pointlike generated e-independent subalgebra coalesces with the gauge invariant subalgebra. So the stringlike approach complements the gauge invariant construction by incorporating the charged sector of QED with its infraparticle aspects and hopefully also the nonlocal aspects of gluons and quarks which are the key to their "invisibility".

Whereas free vectorpotentials have a harmless string localization since by applying a differential operator one can get rid of the semiinfinite string and return to the pointlike field strength, we will see in the next section that the interaction furnishes the charge carrying operators with a much more autonomous stringlike localization which cannot be removed by differential operators and in fact is intrinsic to the concept of electric charge.

The noninteracting  $(0, s = 2)$  representation is usually described in terms of pointlike field strength in form of a 4-degree tensor which has the same permutation symmetries as the Riemann tensor (often referred to as the *linearized Riemann tensor*) with  $d_{sca} = 3$  whereas its string localized covariant potential  $g_{\mu\nu}(x, e)$  has the best possible dimension  $d_{sca} g = 1$ . By allowing string localized potential one can for all  $(m = 0, s \geq 1)$  representations avoid the increase in the dimensions with growing spin in favor of  $d_{sca} = 1$  (independent of spin) stringlike potentials from which one may return to the pointlike field strengths by applying suitable differential operators. In the massive case there is no reason for doing this from the point of localization rather the only physical reason for using the string like counterparts for the pointlike fields is their lower short distance dimensions; again the optimal value is  $d_{sca}=1$  for all spins. Hence candidates for renormalizable interactions in the sense of power counting exist for all spins.

In order to be able to continue with the standard pointlike perturbative formalism one took recourse to the Gupta-Bleuler or BRST gauge formalism. At the end one has to extract from the results of the pointlike indefinite metric calculations the physical data i.e. perturbative expressions in a Hilbert space

In this respect there is a significant conceptual distinction between e.g. classical ED and QED which is masked by the joint use of the same terminology "gauge". Whereas in classical theory the use of the gauge potential simplifies

calculations and leads to interesting connections with the geometry, in particular with the mathematics of fibre-bundles, the quasiclassical treatment of quantum mechanics in a classical external electromagnetic environment leads to the Aharonov-Bohm effect which is usually considered as the physical manifestation of the vectorpotential.

Finally in the quantum field theoretic setting of QED it becomes indispensable since without the minimal coupling of quantum matter to the potential it would be impossible to formulate QED. In this case the pragmatic meaning of the terminology "gauge principle" stands for the continued use of the standard pointlike field formalism of QFT within an indefinite metric setting and the return via gauge invariance to a restricted Hilbert space setting in which the formal pointlike localization is the same as the physical localization. The string-localized approach is strictly speaking not a gauge theoretic formulation in this sense. But neither is the closely related "axial gauge" formulation since the axial potential already lives in a Hilbert space and hence its localization is already physical. Although the clash between pointlike localization and Hilbert space representation continues to hold for the "potentials" of all  $(m = 0, s \geq 1)$  representations, the analog of gauge theory does not exist or is not known. It seems that in those cases there is no "fake" pointlike formalism which can be corrected by a "gauge principle" which then selects the genuinely pointlike observables from the fake objects; in those cases one has to face the issue of string-localized fields right from the beginning.

The next interesting case beyond  $s = 1$ <sup>31</sup> is  $(m = 0, s = 2)$ ; in that case the "field strength" is a fourth degree tensor which has the symmetry properties of the Riemann tensor; in fact it is often referred to as the linearized Riemann tensor. In this case the string-localized potential is of the form  $g_{\mu\nu}(x, e)$  i.e. resembles the metric tensor of general relativity. The consequences of this localization for a reformulation of gauge theory will be mentioned in a separate subsection.

## 6 String-localization of charged states in QED and Schwinger-Higgs screening

In this section some consequences of working with physical<sup>32</sup> i.e. string-localized vector potentials in perturbatively interacting models will be considered. Whereas all charge neutral objects in QED are pointlike generated, this cannot be true for physical charge-carrying operators. From the previous sections we know that the best noncompact localized charged generators are semiinfinite spacelike strings which, as a result of their simultaneous fluctuations in the Minkowski spacetime  $x$  and the spacelike direction  $e$  (3-dim. de Sitter) have improved short distance

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<sup>31</sup>We omit spinor fields, as the zero mass Rarita-Schwinger representation ( $m=0, s=3/2$ ).

<sup>32</sup>Here we do not distinguish between "physical" and "operator in a Hilbert space" i.e. "unphysical" refers to an object in an indefinite metric space. Of course they maybe good reasons to further restrict this terminology within a Hilbert space setting in a more contextual way.

behavior in  $x$ , namely there always exists a potential with  $d_{sca} = 1$  which is the best short distance behavior which the Hilbert space positivity allows for a vectorpotential associated with a field strength of scaling dimension  $d_{sca} = 2$ . The prize to pay for part of the field strength fluctuations having gone into the fluctuation of the string direction  $e$  is the appearance of infrared divergences which require the distribution theoretical treatment of the variable  $e$ ; this problem must be taken care of with special care in perturbative calculations.

In the previous section we learned that the full covariance spectrum (10) for zero mass finite helicity representation can be regained by admitting stringlike fields. The pointlike *field strength* <sup>33</sup> is then connected with the *stringlike potentials* by covariant differential operators. We have presented structural arguments in favor of using stringlike potentials over field strength even in the absence of interactions when Stokes argument is used to rewrite a quantum magnetic flux integral over a surface into an integral over its boundary. However the most forceful argument is that for each spin  $s \geq 1$  there exists always a potential of lowest possible dimension namely  $d_{sca}(\Psi^{(\frac{s}{2}, \frac{s}{2})}(x, e)) = 1$  which is the power-counting prerequisite for constructing renormalizable interactions.

This holds also in the massive case where the covariance for pointlike fields covers the whole spinorial spectrum (10). Whereas the pointlike fields have an  $d_{sca} \geq 1$  which increases with  $s$ , there also exist stringlike fields with  $d_{sca} = 1$  for any  $s$ . The simplest example would be a massive pointlike vector field  $A_\mu(x)$  with short distance dimension  $d_{ssd} = 2$  and a stringlike potential  $A_\mu(x, e)$  with dimension  $d_{ssd} = 1$ . It is only the stringlike potential which has a massless limit.

In this case there is no representation theoretical reason to introduce them (no clash of localization and positivity), rather the only reason for doing this is to meet the power-counting preconditions for renormalizability. Whereas with pointlike fields the power-counting short distance restriction of maximal  $d_{sd}(\text{interaction}) = 4$  only allows a finite number of low spin models, the stringlike situation increases this number to infinite, since now power-counting renormalizable interactions with string-localized potentials of arbitrary high spin exist. For example for the string-localized  $s=2$  symmetric tensor potential  $g_{\mu\nu}(x, e)$  there exist interactions which obey the power-counting condition, but this of course does not mean that specific interesting models with pointlike observables as the Einstein-Hilbert action, are among this larger class of renormalizable candidates. Instead of searching for a gauge principle which singles out pointlike generated observables (e.g. the Riemann tensor  $R_{\mu\nu\kappa\lambda}$ ), the problem one faces is to understand the relation between a coupling dependent law for the change of potentials under the change of string direction  $e \rightarrow e'$  and the form of the pointlike composites.

It was already mentioned that the string-localization has hardly any physical consequences for photons, since even in the presence of interactions the content of the calculated theory can be fully described in terms of linearly related pointlike field strengths. Even the scattering theory of photons in the charge

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<sup>33</sup>We use this terminology in a generalized sense; all the pointlike generators (the only ones considered in [21]) are called field strength (generalizing the  $F_{\mu\nu}$ ) whereas the remaining string-localized generators are named potentials.



zero sector has no infrared problems. However the interaction-induced string-localization of the charged field which is transferred from the vectorpotentials<sup>34</sup> is a more serious matter; it is inexorably connected with the electric charge, and there is no linear operation nor any other manipulation which turns the noncompact localization of charged quantum matter into compact localization. The argument (8) based on the use of the quantum adaptation of Gauss's law shows that the noncompact (at best stringlike) localization nature of generating Maxwell charge-carrying fields is not limited to perturbation theory.

Its most dramatic observable manifestation occurs in scattering of charged particles. As mentioned before, the infrared peculiarities of scattering of electrically charged particles were first noted by Bloch and Nordsiek, but no connection was made with the string-localization which was suggested<sup>35</sup> at the same time by the formula (1) from gauge theory. One reason is certainly that the standard perturbative gauge formalism (which existed in its non-covariant unrenormalized form since the time of the B-N paper) was not capable to address the construction of string-localized physical fields. This is particularly evident in renormalized perturbation theory which initially seemed to require just an adaptation of scattering theory [4], but whose long term consequences, namely a radical change of one-particle states and the spontaneous breaking of Lorentz invariance, were much more dramatic.

These phenomena were incompletely described in the standard perturbation theory of the gauge setting which had no convincing practicable way to extend the requirement of gauge invariance to the charged sectors. In particular the observable part of the scattering formalism culminated in a calculational momentum space recipe for inclusive cross sections; it was not derived in a spacetime setting as the LSZ scattering formalism for interactions of pointlike fields. The spacetime setting in a theory as QFT, for which everything must be reduced to its localization principles, is much more important than in QM where stationary scattering formulations compete with time-dependent ones. As mentioned before Coulomb scattering in QM can be incorporated into any formulation of scattering theory by extracting a diverging phase factor which results from the long range. Noncompact string-localization is a more violent change from pointlike generated QFT than long- versus short range quantum mechanical interactions.

Perturbative scattering (on-shell) processes represented by graphs which do not contain inner photon lines turn out to be independent of the string direction  $e$  i.e. they appear as if they would come from a pointlike interaction<sup>36</sup>. This includes the lowest order Møller- and Bhaba scattering. The mechanism consists in the application of the momentum space field equation to the  $u, v$  spinor wave functions so that from (17) only the  $g_{\mu\nu}$  term in the photon propagator

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<sup>34</sup>Localization of the free fields, in terms of which the interaction is defined in the perturbative setting, is not individually preserved in the presence of interactions; the would be charged fields are not immune against delocalization from interactions with stringlike vectorpotentials.

<sup>35</sup>Localization properties in terms of gauge dependent fields are not necessarily physical.

<sup>36</sup>The time-ordered correlation functions, of which they are the on-shell restriction, are however string-dependent.

survives. The terms involving photon lines attached to external charge lines do however depend on the string directions, and if the scattering amplitudes would exist (they are infrared divergent), they would be  $e$ -independent. The on-shell infrared divergence and the  $e$ -dependence are two sides of the same coin. One expects the photon inclusive cross section to be finite and  $e$ -dependent (at least in the low energy domain). By using the additional resource of  $e$ -smearing one expects for the first time the possible formulation of a large time convergence aiming directly at inclusive cross sections.

In the sequel some remarks on the perturbative use of stringlike vectorpotentials for scalar QED are presented which is formally defined in terms of the interaction density<sup>37</sup>

$$g\varphi(x)^*(\partial_\mu\varphi(x))A^\mu(x,e) - g(\partial_\mu\varphi(x)^*)\varphi(x)A^\mu(x,e) \quad (29)$$

It is also the simplest interaction which permits to explain the Higgs mechanism as a QED charge-screening. The use of string-localized vectorpotentials as compared to the standard gauge formalism deflects the formal problems of extracting quantum data from an unphysical indefinite metric setting to the ambitious problem of extending perturbation theory to the realm of string-localized fields. This is not the place to enter a presentation of (yet incomplete) results of a string-extended Epstein-Glaser approach. Fortunately this is not necessary if one only wants to raise awareness about some differences to the standard gauge approach.

It has been known for a long time that the lowest nontrivial order for the Kallen-Lehmann spectral function can be calculated without the full renormalization technology of defining time-ordered functions. With the field equation

$$(\partial^\mu\partial_\mu + m^2)\varphi(x) = gA_\mu(x,e)\partial^\mu\varphi(x) \quad (30)$$

the two-point function of the right hand side in lowest order is of the form of a product of two Wightman-functions namely the point-localized  $\langle\varphi(x)\varphi^*(y)\rangle = i\Delta^{(+)}(x-y)$  and that of the string-localized vectorpotential (17)

$$\langle A^\mu(x,e)A^\nu(x',e')\rangle \langle\partial_\mu\varphi(x)\partial_\nu\varphi^*(x')\rangle \quad (31)$$

leading to the two-point function in lowest (second) order

$$(\partial_x^2 + m^2)(\partial_{x'}^2 + m^2) \langle\varphi(x)\varphi^*(x')\rangle_{e,e'}^{(2)} \sim g^2 \langle A^\mu(x,e)A^\nu(x',e')\rangle \langle\partial_\mu\varphi(x)\partial_\nu\varphi^*(x')\rangle \quad (32)$$

which is manifestly  $e$ -dependent in a way which cannot be removed by linear operations as in passing from potentials to field strength. One can simplify the  $e$  dependence by choosing collinear strings  $e = e'$ , but the vectorpotential propagator develops an infrared singularity and in general such coincidence limits (composites in  $d=2+1$  de Sitter space) have to be handled with care (although these objects are always distributions in the string direction i.e. can be smeared

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<sup>37</sup>The integral over the interaction density is formally  $e$ -independent.

with localizing testfunctions in de Sitter space); just as the problem of defining interacting composites of pointlike fields through coincidence limits. The infrared divergence can be studied in momentum space; a more precise method uses the mathematics of wave front sets<sup>38</sup>. This simple perturbative argument works for the second order two-point function, the higher orders cannot be expressed in terms of products of Wightman function but require time ordering and the Epstein-Glaser iteration.

Not all functions of the matter field  $\varphi$  are  $e$ -dependent; charge neutral composites, as e.g. normal products  $N(\varphi\varphi^*)(x)$  or the charge density are  $e$ -independent. On a formal level this can be seen from the graphical representation since a change of the string direction  $e \rightarrow e'$  (19) corresponds to an abelian gauge transformation. The divergence form of the change of localization directions together with the current-vectorpotential form of the interaction reduces the  $e$ -dependence of graphs to *vectorpotentials propagators attached to external charged lines* while all  $e$ -dependence in loops cancels by partial integration and current conservation. This is in complete analogy to the standard statement that the violation of gauge invariance and the cause of on-shell infrared divergencies on charged lines result from precisely those external charge graphs; external string-localized vectorpotential lines cause no problems since they loose their  $e$ -dependence upon differentiation. A *neutral external composite* as  $\varphi\varphi^*$  on the other hand does not generate an external charge line; again the gauge invariance argument parallels the statement that such an external vertex does not contribute to the string-localization.

Hence both the gauge invariance in the pointlike indefinite metric formulation and the  $e$ -independence in the string-like potential formulation both lead to pointlike localized subtheories<sup>39</sup>. But whereas the embedding theory (Gupta-Bleuler, BRST) in the first case is unphysical<sup>40</sup>, the string-like approach uses Hilbert space formulations throughout. The pointlike localization in an indefinite metric description is a fake. Its technical advantage is that pointlike interactions, whether in Hilbert space or in a indefinite metric setting, are treatable with the same well known formalism. The gauge invariant correlation define (via the GNS construction) a new Hilbert space which coalesces with the subspace obtained by application of the pointlike generated subalgebra of the physical string-like formulation to the vacuum.

But whereas the noncompact localized charge-carrying fields are objects of a

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<sup>38</sup>Technical details as renormalization, which are necessary to explore these unexplored regions, will be deferred to separte work.

<sup>39</sup>Note however that the spacetime interpretation of the  $e$  is not imposed. The proponents of the axial gauge could have seen in in the free two-pointfunction of vectorpotentials and in all charge correlators if they would have looked at the commutators inside their perturbative correlation functions. The axial "gauge" is not a gauge in the usual understanding of this terminology.

<sup>40</sup>The pointlike localization in an indefinite metric description is a fake. Its technical use is that pointlike interactions, whether in Hilbert space or in a indefinite matric setting, come with a well known formalism. The gauge invariant correlation define (via the KMS construction) a new Hilbert space which coalesces with the subspace obtained by application of the pointlike generated subalgebra of the physical string-like formulation to the vacuum.

physical theory it has not been possible to construct physical charged operators through Gupta-Bleuler or BRST cohomological descent. The difficulty here is that one has to construct non-local invariants under the nonlinear acting formal BRST symmetry. So the simplicity of the gauge formalism has to be paid for when it comes to the construction of genuinely nonlocal objects as charged fields.

This leaves the globally charge neutral *bilocals* in the visor. Their description is expected to be given in terms of formal *bilocals which have a stringlike "gauge bridge"* linking the end points of the formal bilocals (2). In contrast to the string-localized single operators it is difficult to construct them in perturbation theory starting from string-localized free fields, they are too far removed from the form of the interaction (see also next section). In order to understand the relation between such neutral bilocals and infraparticles one should notice that in order to approximate a scattering situation, the "gauge-bridge" bilocals will have to be taken to the limiting situation of an infinite separation distance, so that the problem of the infinite stringlike localization cannot be avoided since it returns in the scattering situation. The only new aspect of the proposed approach based on string-localized potentials which requires attention is that the dependence on the individual string directions  $e$  is distributional i.e. must be controlled by (de Sitter) test function smearing and moreover that composite limits for coalescing  $e$ 's can be defined.

Finally there is the problem of Schwinger-Higgs mechanism in terms of string localization. The standard recipe starts from scalar QED which has 3 parameters (mass of charged field, electromagnetic coupling and quadrilinear selfcoupling required by renormalization theory). The QED model is then modified by Schwinger-Higgs screening in such a way that the Maxwell structure remains and the total number of degrees of freedom are preserved. The standard way to do this is to introduce an additional parameter via the vacuum expectation value of the alias charged field and allow only manipulations which do not alter the degrees of freedom. We follow Steinmann [7], who finds that the screened version consists of a selfcoupled real field  $R$  of mass  $M$  coupled to a vectormeson  $A^\mu$  of mass  $m$  with the following interaction

$$L_{int} = gmA^\mu A_\mu R - \frac{gM^2}{2m}R^3 + \frac{1}{2}g^2 A_\mu A^\mu R^2 - \frac{g^2 M^2}{8m^2}R^4 \quad (33)$$

$$\Psi = R + \frac{g}{2m}R^2 \quad (34)$$

The formula in the second line is obtained by applying the prescription  $\varphi \rightarrow \langle \varphi \rangle + R + iI$  to the complex field within the neutral (and therefore point-local) composite  $\varphi\varphi^*$  and subsequently formally eliminating the  $I$  field by a gauge transformation. The result is the above interaction where  $A_\mu$  and  $R$  are now massive fields. Since the field  $\Psi$  is the image of a pointlike  $\varphi\varphi^*$  under the Higgs prescription, the real matter field  $\Psi$  is point-local.

The important point which formalizes the meaning of "screening" is that the algebraic Maxwell structure as well as the degrees of freedom remain preserved<sup>41</sup>

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<sup>41</sup>The degrees of freedom of the real massive field  $I(x)$  went into the conversion of a photon into a massive vectormeson.

even though the interaction in terms of the new fields  $R$  and the massive vector-potential  $A_\mu(x, e)$  breaks the charge symmetry (by "screening" i.e. trivializing the charge, see below) and the even-odd symmetry  $R \rightarrow -R$  of the remaining  $R$ -interaction. It is this discrete symmetry breaking which renders the even-odd selection rule ineffective and by preventing that  $R$  can have a different localization from  $R^2$  the pointlike localization of the quadratic terms is transferred to the linear  $R$ . The stringlike  $d_{sca} = 1$  massive vectormeson  $A_\mu(x, e)$  played the important technical role in the renormalizability of the theory but is not needed to describe the constructed theory in terms of generating fields: a pointlike  $F_{\mu\nu}$  (or an associated pointlike  $A_\mu(x)$ ) and a pointlike  $R(x)$ .

Hence in the present context the string-localized potentials, as well as the BRST formalism, behave as a "catalyzer" which makes a theory amenable to renormalization. The former have the additional advantage over the latter that the Hilbert space is present throughout the calculation.

One has to be careful in order not to confuse computational recipes with physical concepts. Nonvanishing vacuum expectations (one-point functions) are part of a recipe and should not be directly physically interpreted, rather one should look at the intrinsic observable consequences<sup>42</sup> before doing the physical mooring. The same vacuum expectation trick applied to the Goldstone model of spontaneous symmetry breaking has totally different consequences from its application in the Higgs-Kibble (Brout-Englert, Guralnik-Hagen) symmetry breaking.

In the case of spontaneous symmetry breaking (Goldstone) the *charge associated with the conserved current diverges* as a result of the presence of a zero mass Boson which couples to this current. On the other hand in the Schwinger-Higgs screening situation *the charge of the conserved current vanishes* (i.e. is completely screened) and hence there are no charged objects which would have to obey a charge symmetry with the result that the lack of charge resulting from a screened Maxwell charge looks like a symmetry breaking.

$$Q_{R,\Delta R} = \int d^3x j_0(x) f_{R,\Delta R}(x), \quad f_{R,\Delta R}(x) = \begin{cases} 1 & \text{for } |\mathbf{x}| < R \\ 0 & \text{for } |\mathbf{x}| \geq R + \Delta R \end{cases}$$

$$\lim_{R \rightarrow \infty} Q_{R,\Delta R}^{spon} |0\rangle = \infty, \quad m_{Goldst} = 0; \quad \lim_{R \rightarrow \infty} Q_{R,\Delta R}^{screen} \psi = 0, \quad \text{all } m > 0$$

That the recipe for both uses a shift in field space by a constant does not mean that the physical content is related. The result of screening is the vanishing of a Maxwell charge which (as a result of the charge superselection) allows a copious production of the remaining  $R$ -matter. Successful recipes are often placeholders for problems whose better understanding needs additional conceptual considerations. In both cases one can easily see that the incriminated one-point vacuum expectation has no intrinsic physical meaning, i.e. there is nothing in the intrinsic properties of the observables of the two theories which reveals that a nonvanishing one-point function was used in the recipe for its construction. For a detailed discussion of these issues see [36].

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<sup>42</sup>These are properties which can be recovered from the observables of the model i.e. they do not depend on the particular method of construction.

The premature interpretation in terms of objects which appear in calculational recipes tends to lead to mystifications in particle theory; in the present context the screened charged particle has been called the "God particle". As mentioned before the Schwinger-Higgs screening is analog to the quantum mechanical Debye screening in which the elementary Coulomb interaction passes to the screened large distance effective interaction which has the form of a short range Yukawa potential. The Schwinger-Higgs screening does not work (against the original idea of Schwinger) directly with spinor- instead of scalar matter. If one enriches the above model by starting from QED which contains in addition to the charged scalar fields also charged Dirac spinors then the screening mechanism takes place as above via the scalar field which leads to a loss (screening, bleaching) of the Maxwell charge while the usual charge superselection property of complex Dirac fields remains unaffected.

The Schwinger-Higgs mechanism has also a scalar field multiplet generalization to Yang-Mills models; in this case the resulting multicomponent pointlike localized massive model is much easier to comprehend than its "charged" string-localized origin. As the result of screening there is no unsolved confinement/invisibility problem resulting from nonabelian string-localization.

The Schwinger Higgs screening suggests an important general idea about renormalizable interactions involving massive  $s \geq 1$  fields, namely that formal power-counting renormalizability ( $d_{sdd} = 1$ ) is not enough. For example a pure Yang-Mills interaction with massive gluons (without an accompanying massive real scalar multiplet) could be an incorrect idea because the string-localization of the Hilbert space compatible gluons could spread all over spacetime or there may exist other reasons why the suspicion that such theories are not viable may be correct. Such a situation would then be taken as an indication that a higher spin massive theory would always need associated lower spin massive particles in order to be localizable; in the  $s=1$  case this would be the  $s=0$  particle resulting from Schwinger-Higgs screening. Before one tries to understand such a structural mechanism which requires the presence of localizing lower spin particles it would be interesting to see whether these new ideas allow any renormalizable  $s=\frac{3}{2}$  (Rarita-Schwinger) theories. Even though there may be many formal power-counting renormalizable massive  $s \geq 1$  interactions only a few are expected to be pointlike localized.

It is interesting to mention some mathematical theorems which support the connection between localization and mass spectrum. The support for placing more emphasis on localization in trying to conquer the unknown corners of the standard model comes also from mathematical physics. According to Swieca's theorem<sup>43</sup> [35][36] one expects that the screened realization of the Maxwellian structure is local i.e. the process of screening is one of reverting from the electromagnetic string-localization back to point locality together with passing from a gap-less situation to one with a mass gap. Last not least the charge screening

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<sup>43</sup>Actually Swieca does not use locality directly but rather through its related formfactor analyticity which is different for string-localized (Maxwell) charged particles (less analyticity) from neutral massive screened particles.

leads to a Maxwell current with a vanishing charge<sup>44</sup> and the ensuing copious production of alias charged particles. The loss of the charge superselection rule in the above formulas (33) is quite extreme, in fact even the  $R \leftrightarrow -R$  selection rule has been broken (33) in the above Schwinger-Higgs screening phase associated with scalar QED. The general idea for constructing renormalizable couplings of massive higher spin potentials interacting with themselves or with normal  $s=0,1/2$  matter cannot rely on a Schwinger-Higgs screening picture because without having a pointlike charge neutral subalgebra for zero mass potentials as in QED, which is the starting point of gauge theory, there is no screening metaphor which could preselect those couplings which have a chance of leading to a fully pointlike localized theory, even though renormalizability demands to treat all  $s \geq 1$  as stringlike objects with  $d_{sca}=1$ . Of course at the end of the day one has to be able to find the renormalizable models which maintain locality of observables either in the zero mass setting as (charge-neutral) subalgebras (QED, Yang-Mills) or the massive theories obtained from the former with the help of the screening idea. gauge theory is a crutch whose magic power is limited to  $s=1$ , for  $s > 1$  it lost its power and one has to approach the localization problem directly.

The existence of a gauge theory counterpart, namely the generalization of the BRST indefinite metric formalism to higher spins, is unknown. So it seems that with higher spin one is running out of tricks, hence one cannot avoid confront the localization problem of separating theories involving string-localized potentials which have pointlike generated subalgebras from those which are totally nonlocal and therefore unphysical. This opens a new chapter in renormalization theory and its presentation would, even with more results than are presently available, go much beyond what was intended under the modest title of this paper.

An understanding of the Schwinger-Higgs screening prescription in terms of localization properties should also eliminate a very unpleasant previously mentioned problem which forces one to pass in a nonrigorous way between the renormalizable gauge (were the perturbative computations take place) and the "unitary gauge" which is used for the physical interpretation. The relation between the two remains somewhat metaphoric.

In contradistinction to theories with string-localized electric charge carrying infraparticles and the Schwinger-Higgs screening mechanism one can study models with a discrete gauge groups on the lattice. In the case of a  $\mathbb{Z}_2$  gauge model the result is a rich phase diagram [37] in which also a phase is realized in which massive excitations (Higgs) exist jointly with free string-localized  $\mathbb{Z}_2$ -charges. The authors interpret this as a realization of massive strings in the sense of [43]. The problem of the mathematical control of continuum limits continues unabated.

The screened interaction between a string-localized massive vectorpotential and a real field (33) remains pointlike because the string localization of the massive vectorpotential only serves to get below the power counting limit but does

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<sup>44</sup>Swieca does not directly argue in terms of localization but rather uses the closely related analyticity properties of formfactors.

not de-localize the real matter field; since the pointlike field strength together with the real scalar field generate the theory, the local generating property holds. In an approach based on string-localization there is only one description which achieves its renormalizability by string-localized potentials.

The BRST technology is highly developed, as a glance into the present literature [41] shows. It certainly has its merits to work with a renormalization formalism which starts directly with massive vectormesons [42] instead of the metaphoric "photon fattened on the Higgs one point function". It is hard to think how the BRST technology for the presentation of the Schwinger-Higgs screening model which starts with a massive vectormeson in [41] can be improved. For appreciating this work it is however not necessary to elevate "quantum gauge symmetry" (which is used as a technical trick to make the Schwinger-Higgs mechanism compliant with renormalizability of massive  $s=1$  fields) from a useful technical tool to the level of a new principle.

Besides the Schwinger-Higgs screening mechanism which leads to renormalizable interactions of massive vectormesons with low spin matter, there is also the possibility of renormalizable "massive QED" which in the old days [45] was treated within a (indefinite metric) gauge setting in order to lower the short distance dimension of a massive vectormeson from  $d_{sca} = 2$  to 1, and in this way stay below the powercounting limit. Such a construction only works in the abelian case; for nonabelian interactions the only way to describe interacting massive vectormesons coupled to other massive  $s=0,1/2$  quantum matter is via Higgs scalars in their Schwinger-Higgs screening role. Whereas the local Maxwell charge is screened, the global charges of the non-Higgs complex matter fields are preserved. It seems that Schwingers original idea of a screened phase of spinor QED cannot be realized, at least not outside the two-dimensional Schwinger model (two-dimensional massless QED).

But the educated conjectures in this section should not create the impression that the role of the Schwinger-Higgs screening in the renormalizability of interactions involving selfcoupled massive vectormesons has been completely clarified; if anything positive has been achieved, it is the demystification of the metaphor of a spontaneous symmetry breaking through the vacuumexpectation of a complex gauge dependent field and the tale of "God's particle" which creates the masses of  $s=1/2$  quantum matter. Actually part of this demystification has already been achieved in [41].

This leads to the interesting question whether, apart from the presence of the Higgs particle (the real field as the remnant of the Schwinger-Higgs screening), there could be an intrinsic difference in the structure of the vectormeson. Such a difference could come from the fact that the screening mechanism does not destroy the algebraic structure of the Maxwell equation, whereas an interaction involving a massive vectormeson coming in the indicated way from a S-H screening mechanism and interacting with spinorial matter fields maintains the Maxwell structure. In the nonabelian case this problem does not arise since apparently the Schwinger-Higgs screening mechanism is the only way to reconcile renormalizability with localizability (or a return to physics from an indefinite metric setting).



This raises the interesting question whether renormalizability and pointlike locality of interactions with massive higher spin  $s > 1$  potentials is always related to an associated zero mass problem via an analog of a screening mechanism in which a lower spin field plays the analog of the Higgs field?

Whereas for interactions between spin one and lower spin fields the physical mechanism behind the delocalization of matter (or rather its noncompact re-localization) is to some degree understood, this is not the case for interacting higher spin matter. Stringlike interactions enlarge the chance of potentially renormalizable (passing the power counting test) theories, in fact stringlike potentials with dimension  $d_{sca} = 1$  exist for any spin (hence infinitely many) whereas the borderline for pointlike interaction is  $s = 1/2$  and with the help of the gauge setting  $s = 1$ . Certain interactions, as the Einstein-Hilbert equation of classical gravity probably remain outside the power-counting limit even in the stringlike potential setting, but certain polynomial selfinteractions between the  $g_{\mu\nu}(x, e)$  with  $dim g_{\mu\nu}(x, e) = 1$  may be renormalizable. The existence of free pointlike field strength (in this case the linearized Riemann tensor) indicates that there may be renormalizable interactions which lead to pointlike subalgebras, but the presence of self-couplings modifies the transformation law under a change of  $e$  (19) which now depends on the interaction as it is well-known from the gauge theoretical formulation for Yang-Mills couplings.

One of course does not know whether QFT is capable to describe quantum gravity, but if it does in a manner which is compatible with renormalized perturbation theory, there will be no way to avoid string-localized tensorpotentials even if the theory contains linear or nonlinear related pointlike localized field strength. The trick of gauge theory, by which one can extract pointlike localized generators without being required to construct first the string-localized ones, is a resource which does not seem to exist for higher spins, not even if one is willing to cope with unphysical ghosts in intermediate steps. The most interesting interactions are of course the selfinteractions between  $(m = 0, s > 1)$ . Here one runs into similar problems as with Yang-Mills models (next section). The independence on  $e$ 's of the local observables leads to nonlinear transformation laws which extend that of free stringlike potentials and the non-existence of linear local observables. Although saying this does not solve any such problem, the lack of an extension of the gauge idea to higher spin makes one at least appreciative of a new view based on localization.

There is one important case which we have left out, namely that of massless Yang-Mills theories interaction with massive matter. This will be discussed in the next section.

There are 2 different categories of delocalization: string-localization with and without nontrivial pointlike-generated subalgebras. Generically the coupling of string-localized fields leads to a theory with no local observables. The models of physical interest are those which contain  $e$  independent subfields. For the case at hand the crucial relation is that the change in the string direction can be written as a derivative as in (19).

## 6.1 A note on massive strings

Historically there is a close relation between charge screening, which re-converts string-like generators to pointlike ones, and a powerful theorem [43] which limits the localization of massive charge-carrying generators on nets of observable algebras to be at worse stringlike (pointlike is a special case). In particular there is no usage for generating "branes". The proof uses the connection between smoothness, and analyticity with localization [44]. Such massive strings lead to the same scattering theory as pointlike fields in the situation of a mass gap, even the crossing property of formfactors seems to be the same although more detailed on-shell analytic properties may be different.

A perturbative realization of strings with a mass gap does not exist. But perhaps this is the result of starting with pointlike free fields. As in the Hilbert space formulation using string-like free potentials one should perhaps start with string-localized massive fields which in addition would also widen the possibilities for renormalizable interactions. Of course there is no principle which says that a more fundamental theory as QFT has to jump over a classical cane; there are good indications that Lagrangian (functional integral) QFT only form a thin subset of all physically acceptable QFTs.

## 7 A perturbative signal of "invisibility" and "confinement"?

The reader may wonder why a concept as confinement, which for more than 4 decades has been with us and entered almost every discourse on strong interactions appears in the title of this section in quotation marks. The truth is that these issues are the least understood aspects of gauge theory, and there are even reasons why this terminology may be questionable.

In QM a confinement into a spatial "cage" can be implemented by choosing a confining potential; since the first models for quark-confinement were quantum mechanical, this explains the origin of the terminology. However the conceptual structure of QFT is radically different, and a spatial confinement is not implementable in a setting of QT in which causal localization is the main physical principle, one can at best attribute a metaphoric meaning: confined from becoming on-shell. In fact it was (and still is) one of the conjectures [36] in the 60s that compact localizability together with a limitation on the phase-space degree of freedom<sup>45</sup> would result in "asymptotic completeness" i.e. the property that every state of the theory can be written as a superposition of (generically infinitely many) particle states. In contradistinction to QM this cardinality per unit phase space volume is not finite, but a cardinality which goes beyond nuclearity would also not be acceptable since it contradicts the causal propagation properties of Lagrangian quantization; more precisely it leads to a

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<sup>45</sup>In the old days this property was called "compactness", the modern somewhat stronger version is called "nuclearity". The expression refers to the cardinality of states in a phase space cell being equal to states in the range of a compact or trace-class operator.

violation of the requirement that there should not be more degrees of freedom in the causal completion of a spacetime region  $\mathcal{O}''$  than there were in the original region  $\mathcal{O}$ . Theories in which this violation occurs have very weird properties: a kind of poltergeist effect where additional degrees of freedom which were not present in the initial region  $\mathcal{O}$  are entering "sideways" into  $\mathcal{O}''$ . In case of manipulations outside the Lagrangian formalism as holographic projections to lower dimensional QFTs, as the AdS-CFT correspondence, this warning becomes very relevant [?].

N-particle states, in such a LQP scenario are simply the stable n-fold counter clicks in a coincidence/anticoincidence arrangement in which the spacetime continuum has been cobbled with counters. Here stable means that a counter-registered event of an n-fold excitation at large times does not later turn any more into an m-state  $m \neq n$  with changed velocities. Only in this case of asymptotic stability it makes sense to replace the word "excitation" (of the vacuum) by "particle".

Part of this picture have been proven. It is true that a stable n-fold vacuum excitation state is a tensor product state of n Wigner particles [28][46]. The only known physical counter example, the electrically charged infraparticle states, which are obtained by applying a smeared string-localized charged (Maxwellian charge) operator to the vacuum and studying its asymptotic behavior, contains the mass shell component with vanishing probability<sup>46</sup> and require a conceptually different scattering treatment [47]. "Infraparticles" are most suitably described in terms of "weights" (a kind of singular state which cannot be associated to a state vector), which are directly related to probabilities. Fortunately the Wigner representation theory extends to weights.

No matter how much one compresses the momentum support against the lower value  $p^2 = m^2$ , one never arrives at a state which is not populated by infinitely many photons; one can only control the value of the measuring resolution  $\Delta$ , but there will be always infinitely many photons with energies below  $\Delta$  which escape detection; if we refine our registering precision i.e.  $\Delta \rightarrow 0$ , we do in the end not register anything since the charged particles with sharp mass are not states but "weights" [28]. In other words there is a certain intrinsic lack of precision in registering infraparticles in their states; assuming that the radiation of photons is the only way by which we can measure the presence of charged particles, the infraparticles with small  $\Delta$  would not be perceived, in agreement with the vanishing inclusive cross section for  $\Delta \rightarrow 0$ .

But what happens in case of pure Yang Mills interaction of string-localized gluons which live in a ghost-free physical space? Ignoring the fact that their transformation law under changes of the string direction  $e$  is more complicated (interaction-dependent), one may point to the very nonlinear structure of Yang-Mills potentials which have to play simultaneously the role of the charged particles and the mediators of their interactions. But such a vague metaphoric idea is no substitute for a realistic explanation. It is however not unreasonable to think that there exist strings of different inner tension. For abelian strings describ-

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<sup>46</sup>The Hilbert space of QED does not contain a Wigner particle with the mass of the electron.

ing charged inraparticles the creation of a pair of oppositely charged localized objects with a gauge bridge between them, the probability that (together with real photon clouds) they continue separating to infinity in form of charged inraparticles is larger than that for competing processes of changing into charge neutral outgoing particles is a picture with a high credidility. The analog for nonabelian theories that to the contrary the probability of asymptotic separation is zero. If one adds that the cause of the breaking of the gauge bridge and the conversion into ordinary particles without a bridge is an increase in energy with the size of the bridge we arrived at the standard metaphoric picture. But this cannot be the end of the story because energy (except that of asymptotic particles) is not a well-defined physical concept. In QFT which is solely built on the modular localization principle each property or mechanism must be reduced to this principle.

In view of the known perturbative (off-shell) infrared divergencies of all correlation functions in nonabelian models, there can be no test of this idea within the standard formulation of gauge theory. Passing to the description in terms of string-localized potentials there exists at least a chance. The reason is that a testfunction smearing in the  $e$ 's removes these infrared divergences (= ultraviolet in  $d=1+2$  de Sitter) controls these divergencies and a limiting procedure similar to that in  $x$  for composite fields is expected lead to one  $e$  for each external nonabelian quark or gluon charge. This would be a perturbative off shell description in a bona fide Hilbert space in which one expects to show that there is no nontrivial scattering probability, not in ordinary scattering theory via a spacetime limiting nor in terms of momentum space prescriptions for inclusive cross sections. A construction of a bridged bilocal in terms of which one could exemplify what happens when one enlarges the bridge separation would probably remain out of reach as it is even in the abelian case of QED.

Another problem which should have a solution in this setting is the construction of composite pointlike localized fields. Whereas in the standard setting this cannot even be formulated, the absence of infrared divergence and limiting procedures for coalescend  $e$ 's are expected to play an important role in the classification of  $e$ -independent composites. They would correspond to the composite gauge invariants as e.g. formally  $F^2$  whose construction in terms of infrared divergent correlations is an ill-defined problem. Finally the problem of the occurrence of local gauge groups would be even more demystified as it already became in the work of Stora. This author showed that there is no necessity to prescribe a group in addition to the number of Yang-Mills fields, rather the form of the interaction follows from the number of selfinteracting string-localized potentials and the existence of pointlike generated subalgebras; in the gauge theoretic setting it would be a consequence of the consistency of the nonabelian BRST formalism. Keeping in mind that the BRST symmetry is a formal device, this recognition protects against wrong associations with physical symmetries. All symmetries are at the end of the day related to localization [28], but gauge "symmetries" are most directly related to the principle of QFT than standard inner symmetries.

The closeness of string-localized potentials to the axial gauge should also

dispels the impression that the subject of the present paper is something very speculative and distant. From a pragmatic viewpoint it is nothing else than the attempt to make sense of the so called axial gauge by understanding the origin of its apparent incurable infrared divergences and figuring out how they arise from overlooked localization problems which cause fluctuating string directions. There is certainly nothing more conservative than tracing the infrared divergences to their origin and taming them by controlling the fluctuation which cause them.

There is of course the still open problem of generalizing the Epstein Glaser setting from pointlike to stringlike fields, a formalism which practitioners of QFT hardly pay attention to since its impact on computation is insignificant (renormalization as one goes along), but which in the new context gains in importance. A spreading de-localization of quantum matter which cannot be controlled in terms of string-like fields (but spreads with the growing internal structure of Feynman graphs) would not be acceptable. It is encouraging that the Epstein-Glaser iteration can be shown to work at least if all  $e$  are on a hyperplane [40].

In recent times methods taken from algebraic QFT have been applied to perturbative gauge theory. Whereas in low spin ( $s < 1$ ) QFT these methods lead to a perturbative presentation of QFT, their application to gauge theory only describe part of the theory. In QED the charged particles remain still outside this formalism. Following Bogoliubov's generating time-ordered  $S(f)$  functional formalism<sup>47</sup>, an algebraic formulation in a compact region was defined and its limit to all spacetime discussed (the algebraic adiabatic limit). In this approach one avoids states and correlation functions and uses only operator-algebraic structures; the box dependence is removed by a kind of "algebraic adiabatic limit" [49] and the role of the BRST construction becomes very clear since the encounter with the infrared problems of correlation functions has been shifted to the later task of constructing physical charges states. The problem of states cannot be avoided if one wants to talk about localization, Maxwell charges and their nonabelian counterparts; but the hope is that by separating the algebraic structure from states, it takes on a more amenable form. This approach should in principle permit the calculation of expectation values of renormalized pointlike composites as  $F^2$ , since one expects that their algebraic adiabatic limit exists. But such perturbative calculations do not yet seem to exist.

Forgetting gauge theory for a moment, one may ask as a problem of principle whether the de-localization in QFT can become so strong that an object cannot be registered in a (always local) counter. Having no clues from interacting models one may look at the only Wigner infinite spin representation family [22] for which there are no pointlike generators i.e. all generators are stringlike. But whereas an interacting charged field applied to the vacuum defines a state which can be decomposed with respect to the Poincaré group into a continuum

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<sup>47</sup>This is a functional and not the S-matrix; its connection with the latter requires special conditions (adiabatic limit).

of pointlike generated components (this is not a decomposition in the operator algebra), it seems to be impossible to measure the "piece of the irreducible string state" which is localized inside the compact (or at least quasi-compact [28]) counter. A local change<sup>48</sup> on an irreducible string state leads to problems with the standard view about the measurement process. Even more problematic is the creation of such an object as the result of scattering off ordinary particles.

The appearance of string-localized representations<sup>49</sup> of the third Wigner class (massless infinite spin) in gauge theories is not very plausible, since in a perturbative setting the kind of irreducible representations of the Poincaré group which appear in an interacting theory is believed to be already decided by the zero order input. In other words, it is difficult to conceive of a mechanism in perturbation theory whereby a free gluon potential  $A_\mu^a(x, e)$  (14) acting on the vacuum interacting with itself passes to an object  $A_\mu^a(x, e)_{i.s.}$  whose application to the vacuum contains irreducible infinite spin Wigner components (22). However outside of perturbation theory this may not be true; there exists presently no theorem which excludes the possibility that the application of interacting gluons to the vacuum contains an irreducible infinite spin representation component. Such components are inert, apart from their coupling to gravity (since they carry nontrivial energy-momentum), and therefore may better fit better to dark matter than to gluons/quarks.

Equally implausible is the presence of objects which only exist as composites e.g. a Yang-Mills theory which consists only of  $F^2$  without gluon degrees of freedom. In the perturbative setting this would mean that the elementary degrees of freedom, which in zero order perturbation theory were formally present in the form of (point- or string-localized) free fields, are in fact a fake in that the *physical theory lives on a lesser number of degrees of freedom*, which from the point of view of the original free fields that went into the interaction density, would be considered as composites. The most popular variant of this picture is that the degrees of freedom of the free massive quark matter only served as a kind of "initial ignition" for getting the perturbative interaction going, but that the Hilbert space in which the interaction takes place has only composite pointlike local generators. But the only known mechanism is the theory of superselected locally generated (only pointlike generators) charges [28] according to which one can recreate the charge states from charge splitting and a "disposing the unwanted charge behind the moon" argument [28]. This argument has already its problem with Maxwell charges and fails completely for gluons and quarks.

The most popular semi-phenomenological picture going into this direction is that of globally color neutral *bridged bilocals* quarks which are supposed to break beyond a certain distance and pass to states consisting of physical particles

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<sup>48</sup>In order to be not bothered by vacuum polarization as a result of sharp localization, it is customary to work with quasilocal counters [28].

<sup>49</sup>In section 3 we made a distinction between string-localized representations and zero mass string-localized covariant potentials in pointlike generated (by "field strengths") representations which do not exist as pointlike objects and whose only mark on the representation is the A-B effect.

(hadronization). Here the bridge refers to the localization of a connecting string. From a QFT-based conceptual (as opposed to QM) view such a quark is an ontological chimera, since if the degrees of freedom are not in the Hilbert space why should there exist a mock version up to a certain distance. But as long as one keeps useful phenomenological ideas and the mathematical conceptual content of a QFT apart, there is no problem and the splitting string idea may serve as a useful placeholder of an unsolved problem.

Perhaps it is helpful to remind the reader of this very old construction. The simplest illustration of this idea has been given in the 60s for composites of free fields, namely to split a pointlike composite

$$: A^2(x) : \longrightarrow A(x)A(y) \quad (35)$$

by applying a subtle lightlike limiting procedure [8] to the product  $: A^2(x) :: A^2(y) :$  which makes use of the singularity appearing for lightlike separation. This idea was used in chiral models to show that currents determine bilocals [50]. But there is hardly any experience with this splitting in gauge theories [9]. A successful splitting would of course automatically generate a gauge invariant bridged bilocal. It would be a first step in an extension of the DHR superselection theory to gauge theories

The aim of this paper is to recall unexplored (and not explored) regions in QFT (see its title) and shed new light onto them from the principle of localization. All properties met in QFT models can and must be traced back to this principle, only then one can claim to have understood the problem. It has been shown elsewhere [29] that QFT allows a presentation solely in terms of the "modular positioning" of a finite number of "monads" where a monad stands for the algebraic structure (hyperfinite type III<sub>1</sub> von Neumann) which one meets in the form of localized algebras in QFT. The only reason why this is mentioned here is its Leibnizian philosophical content: the wealth of QFT can be encoded into the abstract positioning of a finite number of copies of one monad into a common Hilbert space. The encoding encloses even spacetime (the Poincaré group representations) and the information about the kind of quantum matter. In other words *relative modular positions in Hilbert space have physical reality, the substrate*<sup>50</sup> *which is being positioned does not*. Modular positioning, modular localization and Poincaré symmetry are inexorably interwoven. This may sound provocative, and certainly no practitioner would adopt or even sympathize with such an extreme standpoint, but it is consistent with everything we know about QFT and it constitutes the biggest difference to QM where none of this is realized. A property encountered in a model of QFT has only been really understood, if it has been traced back to the modular localization principle.

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<sup>50</sup>Modular positioning is the most radical form of relationalism since the local quantum matter arises together with internal and spacetime symmetries. In other words the concrete spacetime ordering is preempted in the abstract modular positioning of the monads in the joint Hilbert space.

## 8 Résumé and concluding remarks

The main aim of this paper is the presentation of old important unsolved problems of gauge theory in a new light. The standard gauge approach to the renormalizable model of QED and its Yang-Mills generalization keeps the pointlike formalism for  $s=1$  vectorpotentials and sacrifices the Hilbert space in intermediate computational steps whereas the new string-localized setting avoids the indefinite metric Gupta-Bleuler/BRST formalism and all the undesired aspects which come with it, as the fake pointlike localization of gauge dependent operators and the impossibility to generate noncompact localized physical operators from fake pointlike fields by implementing gauge invariance (invariance under the BRST "symmetry"). In the case of the Higgs mechanism there is in addition the necessity to pass between the unitary (physical) and the renormalizable gauge. Another motivation comes from the most attractive gauge of gauge theory namely the non-covariant axial gauge which has the attractive property of coming with a Hilbert space representation but has an incurable infrared divergency and for this reason fell out of popularity with practitioners. The new viewpoint consists in realizing that this gauge is not really a gauge in the standard use of the terminology, rather it is a semiinfinite string-localized vectorpotential with variable spacelike string directions which acts in a Hilbert space which transforms covariant ( $e$  plays an important role in the covariance law)

A more profound justification for the use of such comes from the fact that, although certain covariant fields cannot exist in the setting of the Wigner representation theory, the situation changes completely if one allows semiinfinite spacelike string-localized covariant fields  $\Psi(x, e)$  of scale dimension 1, which we summarily called *potentials*, since the vectorpotential is the prime example (for higher  $s$  there are also tensorpotentials). These fields fulfill the correct power counting prerequisite for renormalizability and do not need any power counting lowering BRST formalism, not even in the massive case.

It may be helpful to collect the arguments for the use of those noncompact localized potentials (instead of the pointlike indefinite metric potentials) presented in this paper:

- The gauge theoretic argument why electrically charged operators cannot be compactly localized remains obscure. Although the structural argument based on Gauss's law is rigorous, it does not really explain the delocalization in terms of localization aspects of the interaction.
- Rewriting the quantum magnetic flux through a surface via Stokes theorem into an integral over a pointlike vectorpotential leads to a contradiction with the QFT A-B effect, whereas the use of string-localized vectorpotential removes this discrepancy. Although this rather simple calculation does not instruct how to formulate interactions, it does show that in order to avoid incorrect conclusions about localizations, one must either return to field strengths or work with stringlike instead of pointlike vectorpotentials.



- In most perturbative calculations in the gauge theoretical formalism the condition of gauge invariance in terms of BRST invariance is clearly formulated, but gauge invariant correlation functions of composite operators (not to mention charged correlators) are, as a result of computational difficulties, rarely calculated. In the new approach there is *no gauge conditions* to be imposed, rather the perturbative results are already the physical one.
- The reformulation of the Higgs phenomenon in the Schwinger-Higgs screening setting removes some mysterious aspects of the former and brings it into closer physical analogy with the Debye screening mechanism of QM. Whereas the latter explains how long range Coulomb interactions pass to effective Yukawa potentials, the former describes the more radical change from semiinfinite electrically charged strings with infrared photon clouds to massive Wigner particles associated with pointlike fields. This more radical screening is accompanied by a breaking of the charge symmetry (vanishing charge) and the breaking of the even-oddness symmetry of the remaining real field which makes the screening contribution from the alias charge neutral  $\varphi\varphi^*$  sector (after screening) indistinguishable from that of  $\varphi$ .
- The localization issue in case of Yang-Mills interactions and QCD (as well as for selfinteracting  $s \geq 2$  models) is more involved since the change under string direction is dynamical instead of the kinematical law (19) which follows from Wigner's representation theory. This leads to a much stronger infrared behavior, in fact all spacetime correlators are infrared divergent and only some spacetime independent coupling-dependent functions as the beta function are infrared finite. The new string-localization approach explains this and proposes to take care of the  $e$ -fluctuations which cause the infrared divergences and clarify their role in confinement/invisibility and gauge-bridge breaking (jet formation). This, as well as the still missing presentation of an Epstein-Glaser approach in the presence of string potentials, will be the topic of a separate work<sup>51</sup>.
- The approach based on string-localized potentials does not only replace the gauge setting, which resulted from a resolution of the clash between pointlike localization and quantum positivity with the brute force method of indefinite metric, but it is also meant to be useful for higher spins (example:  $g_{\mu\nu}$  string tensorpotentials) where such a gauge trick is not known. In addition to the avoidance of indefinite metric it also lowers the short distance dimension of pointlike field strength  $s+1$  (for spin  $s$ ) to the lowest value  $d_{sca} = 1$  allowed by unitarity which is the prerequisite of having renormalizable interactions for any spin.

Within the conventional standard terminology of QFT the present project to incorporate string-localized objects into already existing settings (standard model,  $s=2$  "gravitons") would be considered as "nonlocal" QFT. To make QFT

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<sup>51</sup>Jens Mund and Bert Schroer, in progress.

compatible with nonlocality is one of the oldest projects of relativistic QT. Apart from early (pre-renormalization) attempts to modify the quantum mechanical commutation relations to make them more quantum-gravity friendly, the more systematic investigations in QFT started in the 50s with attempts by Christensen and Møller to improve the behavior of interactions in the ultraviolet by spreading interaction vertices in a covariant manner. Later attempts included the Lee-Wick proposal to modify Feynman rules by pair of complex poles and their conjugates. All these models were eventually shown to contradict basic macro-causality properties which are indispensable for their interpretation[51]. There are of course relativistic quantum mechanical theories which lead to a Poincaré-invariant clustering S-matrix [29], but do not fit into the causal localization of the QFT setting.

The more recent interest in nonlocal aspects originated in ideas about algebraic structures (noncommutative QFT) which are supposed to replace classical spacetime as the first step towards "quantum gravity". These attempts usually start from a modification of the quantum mechanical uncertainty relation which, since they involve position operators, strictly speaking does not exist in QFT<sup>52</sup>. The only analog of the uncertainty relation which comes to one's mind is the statement that one can associate to a localized algebra  $\mathcal{A}(\mathcal{O})$  and a "collar" of size  $\varepsilon$  (the splitting distance) which separates  $\mathcal{A}(\mathcal{O})$  from its causal disjoint [28], a localization entropy (or energy)  $Ent(\varepsilon)$  which is proportional to the surface and diverges for  $\varepsilon \rightarrow 0$  in a model-independent manner [53]. Whether such relations between the sharpness of localization and the increase of entropy/energy can be the start of a noncommutative/nonlocal project remains to be seen.

If one can consider this thermal relation as a QFT substitute of an uncertainty relation, it points into a quite different direction than the proposal for a more noncommutative modification of QFT [52]. Namely it looks like an invitation to explore connections between thermodynamics/statistical mechanics, a project which Ted Jacobson has pursued for some time [54]. Unlike algebraic modifications for position operators it has the appealing feature of not having to struggle with problems of frame dependence.

The philosophy underlying the noncommutative approach has been nicely exposed in a recent essay by Sergio Doplicher [55]. His emphasis that a principle as causal localization can only be overcome by another principle which contains the known one in the limiting situation of large distances is certainly well taken, as in many cases, the devil is in the details.

In the present work, the nonlocal behavior remains part of QFT; it may go against certain formalisms as Lagrangian quantization or functional representations, but it certainly does not lead to reasons to change the principles of QFT and it also is not "revolutionary", it only belongs to one of its unexplored corners. Even with respect to quantum gravity the two nonlocal approaches remain different. Within the nonlocality allowed by QFT it would be tempting to relate gravity with selfinteracting  $d_{sca} = 1$  string-localized tensor potentials

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<sup>52</sup>As a result these attempts lead to problems with the principle of independence of the reference frame and in a certain sense open the backdoor for the return of the ether.

$g_{\mu\nu}(x, e)$ . The hope is that one can access this problem (and if necessary dismiss it) with more conventional means.

The aim of this paper was to cast new light on unexplored regions of gauge theory based on recent progress in the understanding of modular localization. There was however no attempt to go into the important details of the new perturbation theory in terms of string-localized potentials. This will be the subject of forthcoming work [40].

*Acknowledgements:* I thank Jakob Yngvason for his invitation to visit the ESI in Vienna and for his hospitality. Part of the research which entered this paper was carried out at the ESI. I also acknowledge several discussions with Jakob Yngvason especially on matters related to section 5.

This work is in a way a continuation of a paper written several years ago together with Jens Mund and Jakob Yngvason. I am indebted to Jens Mund with whom I have maintained close scientific contact and who has kept me informed about recent results about attempts to adapt the Epstein-Glaser renormalization and the idea of wave front sets to string-localized fields. I owe Karl-Henning Rehren thanks for bringing me up to date on matters which were relevant for the appendix.

## 9 Appendix

### 9.0.1 Modular group for conformal algebras localized in doubly-connected spacetime rings

It has been known for a long time that the modular group for a conformal double cone which is placed symmetrically around the origin is related to that of its two-dimensional counterpart by rotational symmetry. In other words if

$$x_{\pm}(\tau) = \frac{(1 + x_{\pm}) - e^{\tau}(1 - x_{\pm})}{(1 + x_{\pm}) + e^{\tau}(1 - x_{\pm})}, \quad -1 < x_{\pm} < 1 \quad (36)$$

represents the two-dimensional conformal modular group in lightray coordinates for a two-dimensional double cone symmetrically around the origin, then the modular group of a symmetrically placed four-dimensional double cone which results from the two-dimensional region by rotational symmetry acts as above by simply replacing  $x_{\pm}(\tau)$  in the above formula by their radial counterpart [56]

$$x_{\pm}(\tau) \rightarrow r_{\pm}(\tau), \quad \varphi, \theta, \tau - \text{independent} \quad (37)$$

The generalization to two and more copies of double cones in two dimensions, symmetrically placed on both sides of the origin is obviously a group which in terms of  $x_{\pm}$  has 4 or  $2n$  fixed points which are the endpoints of two separated intervals. The construction of explicit formulae for  $n$  intervals  $E = I_1 \cup I_2 \dots \cup I_n$  with  $2n$  fixed points is well-known; they are most conveniently obtained as

Caley-transforms of one-parametric subgroups of  $Diff(S^1)$

$$\begin{aligned} f_\tau^{(n)}(z) &= \sqrt[n]{Dil(-2\pi\tau)z^n}, \quad Dil(-2\pi t)x = e^{2\pi t}x \\ x \rightarrow z &= \frac{1+ix}{1-ix}, \quad \text{Cayley transf. } \dot{R} \rightarrow S^1 \end{aligned} \quad (38)$$

This diffeomorphism group in terms of  $x$  is infinity-preserving. By applying further infinity preserving symmetry transformations (translations, dilations) we may achieve the desired symmetric situation with respect to the origin.

For  $n=2$  the two double cones are the  $x$ - $t$  projections of 4-dimensional matter localized in  $\mathcal{T}$  and not matter in a 2-dimensional conformal theory. This suggests that in looking for a geometric analog of (37) one should be aware that the full diffeomorphism group  $Diff(S^1)$  has no analog in 4 dimensions; in fact not even the Moebius subgroup associated to the Virasoro generator  $L_0$  has a counterpart. Hence arguments which are based on properties of  $L_0$  as the necessity to work with split vacua states [57] or with "mixing" [58][33] are not applicable here.

The use of the above formalism in connection with modular theory of multi-intervals and two-dimensional multi-double cones has been presented in detail in [57]. In particular it was shown that in the presence of the  $L_0$  in the Virasoro algebra there is no global representation of the  $f_\tau^{(n)}(z)$  diffeomorphism. Rather the best one can do by choosing instead of the global vacuum the so-called split vacuum is to represent this diffeomorphism group on  $E$  and have a non-geometric action on its complement  $E'$ , or construct a "geometric state" (another split vacuum) for  $E'$  and find a nongeometric action on  $E$ .

In the special case of a chiral Fermion one can achieve a global quasi-geometric action in the vacuum at the expense of a mixing between the different intervals by a computable mixing matrix [58][33]. But only the projections of localized zero mass matter in  $d=1+3$  are candidates for a pure geometric action in their standard vacuum state.

This difference extends to the explanation of violation of Haag duality for  $(m=0, s \geq 1)$ . Whereas in the chiral case this is due to charge transporters whose construction requires the setting of field theory with its characteristic property of vacuum polarization, the Aharonov Bohm effect in QFT (and its higher spin  $s>1$  generalization) can be fully described in the Wigner one-particle representation. It is the only known topological effect in QFT which is of a purely classical origin.

The localization of  $n$   $\mathcal{T}$  symmetrically placed around the origin has a  $x$ - $t$  projection which consists of  $n$  symmetrically arranged two-dimensional double cones. The diffeomorphism group which leaves this figure invariant is a particular diffeomorphism group which in lightray coordinates is a diffeomorphism with  $2n$  fixed points. The number of stringlike potential associated with a pointlike field strength increases with spin  $s$ ; there is always one with the lowest possible dimension which is  $d_{sca}=1$  and the one with the highest dimension has a  $d_{sca}$  which is smaller than that of the lowest field strength. So the A-B fluxes which account for the string-localized potentials are certainly expected to increase with

s. But the situation of  $n$  disconnected  $\mathcal{T}$  appears repetitive. It would be fascinating if the increase of  $s$  could be linked with the occurrence of a new type of A-B effect in higher genus (higher connectivity) analogs of  $\mathcal{T}$  instead of being  $n$ - $\mathcal{T}$  repetitive.

## References

- [1] B. Schroer, *The holistic structure of causal quantum theory, its implementation in the Einstein-Jordan conundrum and its violation in more recent particle theories*, arXiv:1107.1374
- [2] G. Scharf, *Quantum Gauge Theory, A True Ghost Story*, John Wiley & Sons, Inc. New York 2001
- [3] F. Bloch and A. Nordsieck, Phys. Rev. **52**, (1937) 54
- [4] D. Yenni, S. Frautschi and H. Suura, *The infrared divergence phenomena and high energy processes*, Ann. of Phys. **13**, (1961) 370-462
- [5] B. Schroer, Fortschr. Physik **143**. (1963) 1526
- [6] B. Schroer, *Pascual Jordan's legacy and the ongoing research in quantum field theory*, arXiv:1010.4431
- [7] O. Steinmann, *A New Look at the Higgs-Kibble Model*, arXiv:0804.2989
- [8] J. Langerholc and B. Schroer, *Can current operators determine a complete theory ?*, Communications in Mathematical Physics **4**, (1967) 123
- [9] S. Jacobs, *Eichbrücken in der klassischen Feldtheorie*, DESY-THESIS-2009-009
- [10] J. Schwinger, Phys. Rev. **128** (1962), 2425; Theoretical Physics, Trieste Lectures, 1962 p- 89, I.A.E.A, Vienna , 1963
- [11] W. Higgs, Phys. Rev. Lett. **12**, (1964) 132
- [12] G. Lechner, *An Existence Proof for Interacting Quantum Field Theories with a Factorizing S-Matrix*, Commun. Mat. Phys. **227**, (2008) 821, arXiv.org/abs/math-ph/0601022
- [13] Dollard, J. D. J. Math. Phys. **5**, (1964) 729
- [14] D. Buchholz, Phys. Lett. **B174**, (1986) 331
- [15] T. W. B. Kibble, Phys. Rev. **173**, (1968) 1527
- [16] S. Mandelstam, Quantum electrodynamics without potentials, Ann. Phys. **19** (1962), 1-24

- [17] R. Ferrari, L. Picasso and F. Strocchi, Comm. Math. Phys. **35**, (1974) 25; Nuovo Cim. **39** A, (1977) 1
- [18] G. Morchio and F. Strocchi, Nucl. Phys. **B211**, 471, (1983) 232
- [19] J. Fröhlich, G. Morchio and F. Strocchi, Ann. Phys. 119, (1979) 241
- [20] D. Buchholz, S. Doplicher, G. Morchio, J. E. Roberts and F. Strocchi, Annals Phys. 290 (2001) 53
- [21] S. Weinberg, *The Quantum Theory of Fields I*, Cambridge University Press 1995
- [22] J. Mund, B. Schroer and J. Yngvason, *String-localized quantum fields and modular localization*, CMP **268** (2006) 621, math-ph/0511042
- [23] R. Brunetti, D. Guido and R. Longo, *Modular localization and Wigner particles*, Rev.Math.Phys. **14**, (2002) 759
- [24] R. Penrose, Proc. Zero Rest-Mass Fields Including Gravitation: *Asymptotic Behaviour*, R. Soc. Lond. A **23** (1965) 159-203
- [25] J. Mund, *String-Localized Quantum Fields, Modular Localization, and Gauge Theories*, New Trends in Mathematical Physics (V. Sidoravicius, ed.), Selected contributions of the XVth Int. Congress on Math.Physics, Springer, Dordrecht (2009), 495
- [26] J. Yngvason, Commun. Math. Phys. **18** (1970), 195
- [27] S. Weinberg, *Feynman rules for any spin*, Phys. Rev. **133B** (1964) 1318
- [28] R. Haag, *Local Quantum Physics*, Springer, second edition, 1996
- [29] B. Schroer, *Studies in History and Philosophy of Modern Physics* **41** (2010) 104
- [30] P. Leyland, J. Roberts and D. Testard, Duality For Quantum Free Fields, C.N.R.S. preprint July 1978, unpublished
- [31] S. J. Summers, *Tomita-Takesaki Modular Theory*, math-ph/0511034
- [32] Y. Kawahigashi, R. Longo and M. Mueger, Commun. Math. Phys. **219** (2001) 631
- [33] R. Longo, P. Martinetti and K-H Rehren, Geometric modular action for disjoint intervals and boundary conformal field theory, arXiv:0912.1106
- [34] S. Doplicher and J.E. Roberts, Commun. Math. Phys. 131 (1990) 51
- [35] J. A. Swieca, Phys. Rev. D **13**, (1976) 312
- [36] B. Schroer, *Particle physics in the 60s and 70s and the legacy of contributions by J. A. Swieca*, arXiv:0712.0371

- [37] K. Fredenhagen and M. Marcu, Charged states in  $\mathbb{Z}_2$  gauge theories, Commun. Math. Phys. **92**, (1983) 81
- [38] B. Schroer, *Causality and dispersion relations and the role of the S-matrix in the ongoing research*, arXiv:1102.0168
- [39] H. Babujian and M. Karowski, Int. J. Mod. Phys. **A1952**, (2004) 34, and references therein to the beginnings of the bootstrap-formfactor program
- [40] J. Mund and B. Schroer, work in progress
- [41] M. Dütsch, J. M. Gracia-Bondia, F. Scheck and J. C. Varilly, *Quantum gauge models without classical Higgs mechanism*, arXiv:1001.0932
- [42] M. Duetsch and B. Schroer, J.Phys. **A33**, (2000) 4317
- [43] D. Buchholz and K. Fredenhagen, *Locality and the structure of particle states*, Commun.Mat.Phys. **84**, (1982) 1
- [44] D. Buchholz and K. Fredenhagen, *Charge screening and mass spectrum in abelian gauge theories*, Nucl. Phys. B **154**, (1979) 226
- [45] J. H. Lowenstein and B. Schroer, Phys. Rev. D **6**, (1972) 1553
- [46] V. Enss, Commun. Math. Phys. **45**, (1975) 35
- [47] D. Buchholz and M. Porrmann, Ann. Inst. Henri Poincare - Physique théorique **52** (1990), 237
- [48] R. Stora, *Local gauge groups in quantum eld theory: perturbative gauge theories*, talk given at the workshop: "Local quantum physics", Erwin Schrodinger Institute, Vienna, 1997
- [49] M. Duetsch and K. Fredenhagen, Commun. Math. Phys. **203** (1999), 71
- [50] K-H Rehren, Lett. Math. Phys. **30**, (1994) 125
- [51] B. Schroer, Ann. Phys. **319**, (2005) 49
- [52] D. Bahns, S. Doplicher, K. Fredenhagen and G. Piaticelli, *Quantum Geometry on Quantum Spacetime: Distance, Area and Volume Operators*, arXiv:1005.2130
- [53] B. Schroer, *BMS symmetry, holography on null-surfaces and area proportionality of "light-slice" entropy*, to be published in Foundations of Physics, arXiv:0905.4435
- [54] T. Jacobson, Phys.Rev.Lett. **75** (1995) 1260
- [55] S. Doplicher, The principles of locality, Effectivness, fate and challenges, arXiv:0911.5136

- [56] P. Hislop and R. Longo, Commun. Math. Phys. **84**, (1982) 71
- [57] Y. Kawahigashi and R. Longo, Commun. Math. Phys. **257**, (2005) 193
- [58] H. Casini and M. Huerta, Reduced density matrices and internal dynamics for multicomponent regions Class. Quant. Grav. **28**, (2009) 185005